Dexterous Manipulation Through Rolling

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Abstract

Nonholonomic constraints in robotic systems are the source of some difficulties in planning and control; however, they also introduce interesting properties that can be practically exploited. In this paper we consider the design of a robot hand that achieves dexterity (i.e., the ability to arbitrarily locate and reorient manipulated objects) through rolling. Some interesting issues arising in planning and controlling motions of such device are considered, including exact planning for a spherical object and approximate planning for general objects. An experimental prototype of a three-plus-one d.o.f. hand achieving dexterous manipulation capabilities is described along with experimental results from manipulation.

1 Introduction

Dexterous hands, i.e. cooperating multilimb robots with the capability of manipulating an object so as to arbitrarily steer its configuration in space, have attracted much interest in the robotics literature. However, the high degree of sophistication in their mechanical design prevented dextrous robotics hand to succeed in applications where factors such as reliability, weight, small size, or cost, were at a premium. One figure partially representing such complicacy is the number of actuators, that ranges between 9 and 32 for typical hands. In this paper, we consider the exploitation of the effects of rolling of the object between the fingers as a means of achieving dexterity while reducing the number of necessary actuators in the hand.

Rolling between rigid bodies in three-dimensional space is a well-known case of nonholonomically constrained motion. A knife-edge cutting a sheet of paper and a cat falling onto its feet are examples of natural nonholonomic systems, while bycicles and cars (possibly with trailers) are familiar examples of artificially designed nonholonomic devices. The most notable characteristic of nonholonomic systems is that they can be driven to a desired configuration in a d-dimensional configuration manifold using less than d inputs. Since "inputs" in engineering terms translates into "actuators", devices designed by intentionally introducing nonholonomic mechanisms can spare hardware costs without sacrifying dexterity. While nonholonomy in a system is often regarded as an annoying side-effect of other design considerations (this is how most people consider e.g. car maneuvering for parallel-parking), purposeful introduction of nonholonomy in robotic system design has been considered previously by Brockett [3], and, in a spirit closer to that of the present paper, by Sordalen and Nakamura [17].

Nonholonomic systems do have disadvantages, however, among which the most notable are perhaps the difficulties in planning and controlling their motions. Planning finger movements to steer an object between an initial and a final desired configuration is not trivial, and in most cases the task is beyond common human ability. This is particularly true when the shape of the object is not known a priori, but has to be reconstructed from sensory data during manipulation. This paper is devoted to describing tools that may render the design of a nonholonomic dextrous hand a viable means of achieving dexterity with simple mechanical design.

2 Background

We recall some basic definitions and facts that are necessary to understand the techniques used in the paper. We will deal with mechanical systems whose configurations evolve in a d-dimensional manifold $\mathbf{M}$, i.e. a differentiable variety locally diffeomorphic to $\mathbb{R}^d$. To avoid unnecessary complication, we will be only concerned here with local representations of the system, so that local coordinates in $\mathbb{R}^d$ are assumed throughout. According to the classical definition of nonholonomy, a system described by its generalized coordinates $\mathbf{q} \in \mathbb{R}^d$ is called nonholonomic if it is subject to constraints of the type

$$\mathbf{c}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) = 0$$

and if there is no equation of the form $\mathbf{c}'(\mathbf{q}(t)) = 0$ such that $\frac{d\mathbf{c}'(\mathbf{q}(t))}{dt} = \mathbf{c}(\mathbf{q}(t), \dot{\mathbf{q}}(t))$. If also Pfaffian (as is our case), the constraint is linear in $\dot{\mathbf{q}}$,

$$\mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{A}(\mathbf{q})\, \dot{\mathbf{q}} = 0,$$

and hence it can be rewritten in terms of a basis of the kernel of $\mathbf{A}(q)$, denoted by $\mathbf{G}(q)$, as

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\, \mathbf{u}$$

This is the standard form of a nonlinear, driftless control systems. In the related vocabulary, components

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of $\mathbf{u}$ are *inputs*, while columns of $\mathbf{G}(\mathbf{q})$ are *input vector fields.* The collection of the subspaces spanned by $\mathbf{G}(\mathbf{q})$ at every $\mathbf{q} \in \mathbf{U} \subset \mathbf{M}$ is a *distribution.* A distribution is nonsingular if $\mathsf{rank}\,(\mathbf{G})$ is constant over its domain. It is involutive if the Lie–Bracket between any of its vector fields is again a vector field in the distribution, i.e. if

$$
\begin{aligned}
[\mathbf{G}_i, \mathbf{G}_j] \stackrel{def}{=} & \frac{\partial \mathbf{G}_j}{\mathbf{q}} \mathbf{G}_i - \frac{\partial \mathbf{G}_i}{\mathbf{q}} \mathbf{G}_j \\
\in & \ \mathsf{span}\,(\mathbf{G}(\mathbf{q})), \forall i, j.
\end{aligned}
$$

In terms of input vector fields, the nonintegrability of the original mechanical constraint has its counterpart in the well-known Frobenius theorem:

Theorem 1 (Frobenius). *A nonsingular distribution is integrable if and only if it is involutive.*

If a distribution is not involutive, then motions in the Lie–bracket directions are possible which are not in the span of the original vector fields. Hence, an alternative viewpoint on nonholonomy is that a system $\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\mathbf{u}$ is nonholonomic if $\mathbf{G}(\mathbf{q})$ is not involutive. A fundamental question at this point is, under what conditions can a d–dimensional nonholonomic system be steered by less than d inputs to an arbitrary configuration. Note that also higher order Lie–brackets represent directions of possible motion. Therefore, one is naturally led to consider the *filtration*

$$
\begin{aligned}
\Gamma_0 &= \ \mathsf{span}\,(\mathbf{G}_i) \\
\Gamma_1 &= \Gamma_0 + [\Gamma_0, \Gamma_0] \\
&= \ \mathsf{span}\,(\mathbf{G}_i, [\mathbf{G}_j, \mathbf{G}_k]); \\
&\ \vdots \\
\Gamma_k &= \Gamma_{k-1} + [\Gamma_{k-1}, \Gamma_0] (\text{brackets of order} \leq k-1);
\end{aligned}
$$

The construction stops at some level, say $k = p$, when $\dim \Gamma_{k+1} = \dim \Gamma_k$. The number k is called "degree of nonholonomy". Let $\dim(\Gamma_i) = \gamma_i$ be constant in some open set. If $p > 0$ and $\gamma_p = n$, the system is completely controllable and it is said "maximally nonholonomic" (Chow's theorem). Vectors $\gamma = [\ \gamma_0 \ \ \gamma_1 \ \ \cdots \ \ \gamma_p \ ]^T$ and $\bar{\gamma} = [\ \gamma_0 \ \ \gamma_1 - \gamma_0 \ \ \cdots \ \ \gamma_p - \gamma_{p-1} \ ]^T$, are called "growth vector" and "relative growth vector", respectively.

The planning problem, i.e. to explicitly find a control $\mathbf{u} : [0,1] \to \mathbb{R}^m$ that steers the nonholonomic system $\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\mathbf{u}(t)$ from given $\mathbf{q}(0)$ to an arbitrary $\mathbf{q}(1)$, has been given much attention in the literature recently. Murray and Sastry [13] investigated a class of systems for which a normal, or "chained" form, of system equations can be obtained, and showed that optimal inputs (in a certain sense) for systems in this form are sinusoids and cosinusoids at integrally related frequencies. Their method, along with extensions made by Sordalen [16], solved the problem of parking cars with an arbitrary number of trailers. On the other hand, Rouchon *et al.* [14] showed that "differentially flat" systems can be conveniently planned looking at their "flat" outputs only. Monaco and Normand–Cyrot [10] proposed to apply nonlinear multirate control to the planning problem for systems that admit an exact sampled model (while maintaining controllability under sampling). Lafferriere and Sussman [8] described a powerful "constructive" method for exactly steering nilpotent systems, i.e. systems whose higher–level Lie–brackets are identically null.

We describe now some tools of surface geometry necessary to deal with our specific problem of manipulation by rolling. Both the object and finger surfaces are assumed to be simple surfaces Σ embedded in $\mathbb{R}^3$, to which coordinate patches $(\mathbf{f}, U); \mathbf{f} : U \subset \mathbb{R}^2 \to \Sigma_U \subset \Sigma$, can be locally attached so as to form an atlas. In these coordinates, a Gauss (normal) map $n : \Sigma \to S^2 \subset \mathbb{R}^3$, can be written as $n = \frac{\mathbf{f}_u \times \mathbf{f}_v}{\|\mathbf{f}_u \times \mathbf{f}_v\|}$). It is also useful to define a normalized Gauss frame $[x, y, z] = [\mathbf{f}_u/\|\mathbf{f}_u\|, \mathbf{f}_v/\|\mathbf{f}_v\|, n]$, with $\mathbf{f}_u^T \mathbf{f}_v = 0$. The kinematics of rolling motions can be derived from either the classical differential geometric viewpoint (using first and second fundamental forms for Σ at p, I_p and II_p resp., and Christoffel symbols of the first and second kind, $[ij, k]$ and Γ_{ij}^k); or using Cartan's definitions of metric form $\mathbf{M}_\Sigma = \ \mathsf{diag}\,(\|\mathbf{f}_u\|, \|\mathbf{f}_v\|)$, curvature form $\mathbf{K}_\Sigma = [\mathbf{x}, \mathbf{y}]^T [\mathbf{z}_u, z_v] \mathbf{M}_\Sigma^{-1}$, and torsion form $\mathbf{T}_\Sigma = \mathbf{y}^T [\mathbf{x}_u, \mathbf{x}_v] \mathbf{M}_\Sigma^{-1}$. While the latter description results more convenient, we recall that the relationship between the two sets of forms is given by $M_\Sigma = \sqrt{I_p}$, $K_\Sigma = M_\Sigma^{-T} II_p M_\Sigma^{-1}$, and $\mathbf{T}_\Sigma \mathbf{M}_\Sigma = \mathbf{M}_{22} \mathbf{M}_{11}^{-1} [\Gamma_{11}^2, \Gamma_{12}^2]$ (cf. e.g. Sarkar [15]).

The kinematic equations of motion of the contact points between two bodies rolling on top of each other describe the evolution of the (local) coordinates of the contact point on the finger surface, $\alpha_f \in \mathbb{R}^2$, and on the object surface, $\alpha_o \in \mathbb{R}^2$, along with the (holonomy) angle between the x–axes of the two gauss frames ψ, as they change according to the rigid relative motion of the finger and the object described by the relative velocity $\mathbf{v}$ and angular velocity ω. According to the derivation of Montana [11], in the presence of friction (soft–finger contact model) one has

$$
\begin{aligned}
\dot{\alpha}_f &= \mathbf{M}_f^{-1} \mathbf{K}_r^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix}; \qquad (1) \\
\dot{\alpha}_o &= \mathbf{M}_o^{-1} \mathbf{R}_\psi \mathbf{K}_r^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix}; \\
\dot{\psi} &= \mathbf{T}_f \mathbf{M}_f \dot{\alpha}_f + \mathbf{T}_o \mathbf{M}_o \dot{\alpha}_o;
\end{aligned}
$$

where $\mathbf{K}_r = \mathbf{K}_f + \mathbf{R}_\psi \mathbf{K}_o \mathbf{R}_\psi$ is the relative curvature form, and

$$
\mathbf{R}_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}.
$$

3 Hand Kinematics

To completely describe the manipulation system, we need to attach the rolling equations above to the kinematics of the manipulating hand by means of the constraint equations imposed by the no-slippage condition. Let $\mathbf{c}_i \in C_i \subset SE(3)$ describe the position and orientation w.r.t. base frame of the Gauss frame at the contact between the i-th finger and the object. Denoting by $\mathbf{q}_i \in Q_i \subset \mathbb{R}^{q_i}$ the joint coordinates and by $\alpha_{f_i} \in U_i \subset \mathbb{R}^2$ the contact coordinates for finger i, let $\Lambda_i : U_i \times Q_i \rightarrow C_i$ represent the translational part of $\mathbf{c}_i(\mathbf{q}_i, \alpha_{f_i})$, and $\mathbf{n}_i(\mathbf{q}_i, \alpha_{f_i})$ the normal map at the i-th contact point. For the contact pair between the object and the i-th finger, soft-finger frictional constraints impose that

$$\frac{\partial \Lambda_i}{\partial \mathbf{q}_i}\dot{\mathbf{q}}_i = {}^b\mathbf{v} + {}^b\omega \times \Lambda_i, \qquad (2)$$
$$\mathbf{n}_i^T\, {}^\omega\mathbf{J}_i \dot{\mathbf{q}}_i = \mathbf{n}_i^T\, {}^b\omega$$

where ${}^b\mathbf{v}$, ${}^b\omega$ are the velocity and angular velocity of the object in a base reference frame, and ${}^\omega\mathbf{J}_i(\mathbf{q}_i, \alpha_{f_i})$ represents the rotational Jacobian operator mapping joint velocities $\dot{\mathbf{q}}_i$ in the angular velocities of the contact Gauss frame on the i-th finger. Note that (3) is obtained by equating constrained velocities of the contact point as part of the finger and of the object respectively, and cancelling out the contributions of rolling (terms in $\dot{\alpha}_i$, $\dot{\alpha}_o$) that have already been taken into account in deriving the equations of rolling kinematics. Introducing the notation

$$\mathbf{J}_i = \begin{bmatrix} \frac{\partial \Lambda_i}{\partial \mathbf{q}_i} \\ \mathbf{n}_i^T\, {}^\omega\mathbf{J}_i \end{bmatrix}, \quad \mathbf{G}_i^T = \begin{bmatrix} \mathbf{I} & -\Lambda_i \times \\ 0 & \mathbf{n}_i^T \end{bmatrix},$$

and constructing a global hand "jacobian" matrix $\mathbf{J} = \mathbf{diag}\,(\mathbf{J}_i)$ and "grasp" matrix $\mathbf{G} = [\mathbf{G}_1\ \mathbf{G}_2\ \cdots]$, the hand kinematics can be written as

$$\mathbf{J}(\mathbf{q}, \alpha_{f_1}, \alpha_{f_2}, \ldots)\,\dot{\mathbf{q}} = \mathbf{G}^T(\mathbf{q}, \alpha_{f_1}, \alpha_{f_2}, \ldots)\,{}^b\dot{\mathbf{u}} \qquad (3)$$

where $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_1^T\ \dot{\mathbf{q}}_2^T\ \cdots]^T$, and ${}^b\dot{\mathbf{u}} = [{}^b\mathbf{v}^T\ {}^b\omega^T]^T$. One further step is necessary to relate joint motions to the relative velocities between the object and one of the fingers, used as a reference member. This involves expressing ${}^b\dot{\mathbf{u}}$ in terms of the sum of the velocity of the reference member and of the relative velocity $\dot{\mathbf{u}}$, and bringing the former part to the right hand side of (3). Having modified the hand Jacobian matrix accordingly, the hand kinematics equations (dropping arguments for simplicity) reads as

$$\tilde{\mathbf{J}}\dot{\mathbf{q}} - \mathbf{G}^T\,\dot{\mathbf{u}} = \mathbf{0} \qquad (4)$$

Joint motions can be easily solved in terms of object motions if the hand Jacobian is invertible. However, in order for this condition to apply, it is necessary that the hand has at least four joints per finger. Note that, in the design of a hand system intended to exploit rolling to reduce the number of actuators, the hand Jacobian is certainly not invertible (i.e., the hand is kinematically defective). The kinematics of defective hands have been studied by Bicchi, Melchiorri, and Balluchi [2]. Using their terminology and methods, and assuming that the system is graspable and not

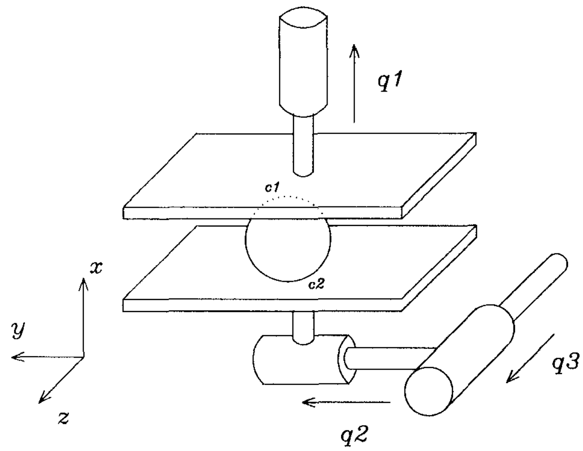

Figure 1: Kinematics of the hand developed for experimenting dextrous manipulation by rolling.

redundant, one can evaluate two matrices $\mathbf{U}_p$ and $\mathbf{Q}_p$ such that their columns span the subspaces of compatible object and joint velocities, respectively. In these hypotheses, there is a bijection between relative velocities $\dot{\mathbf{u}} \in$ **range** $(\mathbf{U}_p)$ and joint velocities $\dot{\mathbf{q}} \in$ **range** $(\mathbf{Q}_p)$, which can be expressed as

$$\dot{\mathbf{q}} = \mathbf{Q}_p\mathbf{U}_p^+\dot{\mathbf{u}}, \quad \dot{\mathbf{u}} \in \textbf{range}\,(\mathbf{U}_p). \qquad (5)$$

Note that, by construction, only the ω_x, ω_y components of $\dot{\mathbf{u}} \in$ **range** $(\mathbf{U}_p)$ result nonzero. Elements of the matrix $\mathbf{Q}_p\mathbf{U}_p^+$ are functions of finger configurations $\mathbf{q}$ and of contact coordinates on all fingers α_{f_i}.

Example 1. The kinematic structure of the hand realized in our laboratory for studying dexterity through rolling is depicted in fig.1

For the plane surface of fingers, described in Cartesian coordinates, the forms involved in the equations of rolling are $\mathbf{M}_p = \mathbf{I}_2$, $\mathbf{K}_p = \mathbf{O}_{2\times 2}$, and $\mathbf{T}_p = \mathbf{O}_{2\times 1}$. For a spherical object of radius R, in spherical coordinates, one has $\mathbf{K}_p = R^{-1}\mathbf{I}_2$, and

$$\mathbf{M}_s = \begin{bmatrix} R & 0 \\ 0 & R\cos(u_s) \end{bmatrix};$$
$$\mathbf{T}_s = \begin{bmatrix} 0 & -R^{-1}\tan(u_s) \end{bmatrix}.$$

Using notation as described by fig.1, the finger kinematics are written as

$$\Lambda_1 = \begin{bmatrix} q_1 \\ \alpha_{f_1,1} \\ \alpha_{f_1,2} \end{bmatrix}; \quad \mathbf{J}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; {}^\omega\mathbf{J}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

and

$$\Lambda_2 = \begin{bmatrix} 0 \\ \alpha_{f_2,1} + q_2 \\ \alpha_{f_2,2} + q_3 \end{bmatrix}; \quad \mathbf{J}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; {}^\omega\mathbf{J}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

Note that for a sphere between two parallel fingers the vector $\Lambda_1 - \Lambda_2$ is constant and equal to $-2R$. Using (2) and (3), and renaming contact coordinates on the lower finger as x, y, and on the sphere as u, v, one gets (on an open subset of the state space not containing $v = \cos^{-1}(0)$):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \\ \dot{\psi} \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ R \\ \frac{\cos\psi}{\cos v} \\ -\sin\psi \\ \tan v \cos\psi \\ 0 \\ 0 \\ 2R \end{bmatrix} \omega_x + \begin{bmatrix} R \\ 0 \\ -\frac{\sin\psi}{\cos v} \\ -\cos\psi \\ \tan v \sin\psi \\ 0 \\ 2R \\ 0 \end{bmatrix} \omega_y. \tag{6}$$

where the contact coordinates on the upper finger finger are trivially obtained and not reported for brevity.

4 Planning Manipulation by Rolling

In this section we will discuss the planning problem for a hand system such as that described in fig.1, with particular reference to the manipulation of a sphere. Although this is perhaps the simplest case, it still provides important insight in the more general problem of manipulating objects of an arbitrary shape.

Considering the filtration associated with system (6), one easily finds that the growth vector for this system is [2, 3, 5, 5, $\cdots$], whence it follows that the manipulation system is not controllable as a whole. However, from closer inspection, it turns out that system (6) can be effectively decoupled in the upper 5–dimensional part (kinematics of the object rolling on a plane finger), which is controllable, and the lower 3–dimensional part (hand kinematics). Therefore, an arbitrary change of position and orientation of a sphere can be achieved by the hand of fig.1, provided that an additional rigid translation of the hand–object system as a whole can be actuated by a fourth motor (not shown in fig.1). The final position of fingers will not be controllable. For more general cases, the only results (to our knowledge) are those of Li and Canny [9], showing that controllability is lost in the rolling of a sphere on top of another sphere only when the radii are coincident or either of them vanishes. Motivated by these results, it seems reasonable the ***conjecture*** that controllability of rolling motions between surfaces is generic. Note also that, in the hand shown in fig.1, actuation of joint 1 is only necessary in order to maintain contact and prevent slippage between surfaces, which goal could be in principle realized by using passive devices (e.g., preload springs). According to the conjecture above, and recalling our previous definition of a dextrous hand as a device capable of arbitrarily positioning and orienting the object, a general remark can be stated as:

a dexterous hand can be built in principle by using only three actuators.

The study of the rolling motion of a sphere on a plane is a classical problem in rational mechanics, recently brought to the attention of the control community by Brockett and Dai [4], who provided optimal planning solutions for an approximated version of the problem. Jurdjevic [7] investigated optimal solutions of the original problem and showed its relationship with the clasical problem of the ***elastica***. Li and Canny [9] proposed a planning algorithm based on the use of the Gauss–Bonnet theorem in differential geometry, obtaining an elegant algorithm capable of bringing the sphere to the desired position and orientation by a sequence of three movements. However, these techniques are special to the case of a spherical object, and there is no clue as to how they could generalize to arbitary surfaces.

In the broader repertoire of planning methods for nonholonomic systems, effective planning algorithms are available for systems that can be put in a convenient form. However, it can be shown, based on the fact that the relative growth vector of the system (6), i.e. $\tilde{\gamma} = [2\ 1\ 2]^T$, that it cannot be put in chained form, nor it is differentially flat (see Murray [12]). On the other hand, system (6) is not in nilpotent form, so that application of the constructive method of Lafferriere and Sussmann [8] would only provide approximate results. Furthermore, direct application of multirate digital control techniques to the system (6) is not possible, since the corresponding exact sampled model is not available.

Notwithstanding the genericity of its growth vector, the controllable part of the kinematic equations of manipulation does possess a structure that can be exploited to find efficient planning algorithms. An useful result in this sense is the following, holding for arbitrary surfaces rolling on a planar finger:

Proposition 1 *(Bicchi and Sastry, 1994).* ***Assuming that either surface in contact is (locally) a plane, there exist a state diffeomorphism and a regular static state feedback law such that the kinematic equations of contact (2) assume a strictly triangular structure.***

Proof. Rewrite (2) as

$$\begin{aligned} \dot{\alpha}_f &= \mathbf{M}_f^{-1}\mathbf{K}_r^{-1}\mathbf{w}; \\ \dot{\alpha}_o &= \mathbf{M}_o^{-1}\mathbf{R}_\psi\mathbf{K}_r^{-1}\mathbf{w}; \\ \dot{\psi} &= [\mathbf{T}_f\mathbf{R}_\psi + \mathbf{T}_o]\mathbf{K}_r^{-1}\mathbf{w}, \end{aligned} \tag{7}$$

where $\mathbf{w}^T = [-\omega_y\ \omega_x]$. Recall that for plane fingers, $\mathbf{T}_f = [0\ 0]$, and $\mathbf{M}_f = \mathbf{I}_2$. Define the regular state feedback $\mathbf{w} = \beta(\alpha_f, \alpha_o, \psi) + \gamma(\alpha_f, \alpha_o, \psi)\hat{\mathbf{w}}$ as

$$\begin{aligned} \beta(\alpha_f, \alpha_o, \psi) &\equiv 0; \\ \gamma(\alpha_f, \alpha_o, \psi) &= \mathbf{K}_r\mathbf{M}_o\hat{\mathbf{w}}, \end{aligned} \tag{8}$$

and apply a change of coordinates that suitably reorders the states, to obtain

$$\begin{aligned} \dot{\alpha}_o &= \hat{\mathbf{w}}; \\ \dot{\psi} &= \mathbf{T}_o\mathbf{M}_o\hat{\mathbf{w}}; \\ \dot{\alpha}_f &= \mathbf{R}_\psi\mathbf{M}_o\hat{\mathbf{w}}, \end{aligned}$$

which is strictly lower triangular. □

As an instance of application of this technique, consider again the case of a spherical object on a planar finger, (6). The state feedback law

$$\mathbf{w} = \begin{bmatrix} \cos v \, \cos\psi & -\sin\psi \\ -\cos v \, \sin\psi & -\cos\psi \end{bmatrix} \hat{\mathbf{w}} \tag{9}$$

transforms (6) in

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \sin v \\ -R\sin\psi\,\cos v \\ R\cos\psi\,\cos v \\ 0 \\ -2R\sin\psi\,\cos v \\ 2R\cos\psi\,\cos v \end{bmatrix} \hat{w}_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -R\cos\psi \\ -R\sin\psi \\ 0 \\ -2R\cos\psi \\ -2R\sin\psi \end{bmatrix} \omega_z . \tag{10}$$

The relevance of strictly triangular forms to planning is in the relative ease by which the flows of the vectorfields can be obtained. In our example, in fact, one has (for the part concerning contact variables)

$$\Phi_t^{g_1} = \begin{bmatrix} u_0 + t \\ v_0 \\ \psi_0 + t\sin v_0 \\ x_0 + \frac{R}{\tan v_0}\left[\cos(\psi_0 + t\sin v_0) - \cos\psi_0\right] \\ y_0 + \frac{R}{\tan v_0}\left[\sin(\psi_0 + t\sin v_0) - \sin\psi_0\right] \end{bmatrix}, \tag{11}$$

and

$$\Phi_t^{g_2} = \begin{bmatrix} u_0 \\ v_0 + t \\ \psi_0 + t\sin v_0 \\ x_0 - Rt\cos\psi_0 \\ y_0 - Rt\sin\psi_0 \end{bmatrix}. \tag{12}$$

A solution to the planning problem for system (6) can now be applied, which is closely related to the multirate technique, and consists in concatenating a sequence of constant inputs of the form $\{\hat{w}_1 = 1, \hat{w}_2 = 0, 0 < t < T_1\}$, $\{\hat{w}_1 = 0, \hat{w}_2 = 1, T_1 < t < T_2\}$, $\ldots\{\hat{w}_1 = 1, \hat{w}_2 = 0, T_4 < t < T_5\}$. The 5 unknown variables T_i can be evaluated by solving the system of five nonlinear equations obtained by equating the final to the desired configuration, namely

$$\Phi^{g_1}_{T_5-T_4} \circ \Phi^{g_2}_{T_4-T_3} \circ \Phi^{g_1}_{T_3-T_2} \circ \Phi^{g_2}_{T_2-T_1} \circ \Phi^{g_1}_{T_1}(\mathbf{x}_0) - \mathbf{x}_{des} = 0 \tag{13}$$

Equivalently, one can fix time interval lengths and vary the amplitude of inputs. Also, allowing a finer discretization of the time scale, other concerns such as minimizing the length of the path or avoiding limits of the workspace can be taken into account by building a suitable optimization problem constrained by (13).

Results of application of this technique to planning the manipulation of a sphere so as to realize a rotation of 30 deg. about the vertical direction, while bringing the contact points on the sphere and on the object back to the original position, are reported in fig.2. The lower left diagram shows the path to be followed by the contact point on the finger surface.

5 Experimental

A nonholonomic dextrous hand with three actuators has been built in our laboratory according to the scheme of fig.1. Joints are actuated by three D.C. motors, and position are sensed by linear potentiometers. One important feature of the hand is that its upper finger is equipped with an intrinsic tactile sensor ([1]) which provides real–time sensing of the actual position of the contact point on the finger. Exploiting the capability of intrinsic tactile sensing to provide also the direction of the contact force (including tangential components), the system is also able to detect the contact point position on the lower finger. The vertical axis is controlled so as to maintain a suitable level of contact force on the object, to avoid slippage. Active grasp force control is particularly important when manipulating objects whose surface is not spherical.

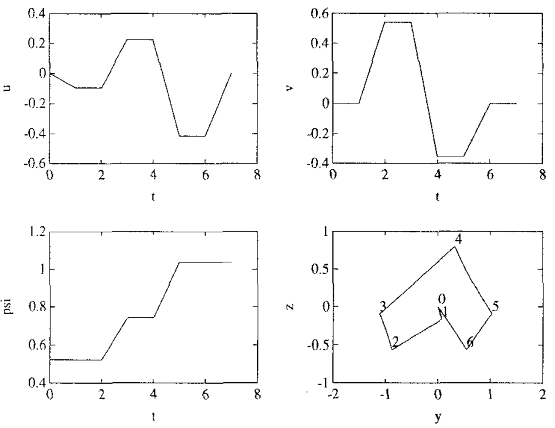

Figure 2: Planned trajectories of the contact coordinates for a rotation of 30 deg., obtained with a sequence of 7 piecewise constant inputs

The main problem in realizing planned manipulations is related to the fact that control inputs $\hat{\mathbf{w}}$ used for planning are not the same as the physical inputs actually available to the controller, that is, joint velocities $\dot{\mathbf{q}}$. While it is possible to integrate (10) so as to obtain the desired joint trajectory for the sphere, the same is much more complex for objects of general shape. Moreover, such process results in a completely open–loop control scheme that is prone to a number of errors in practical implementation. The approach we followed tends to exploit the possibility of using tactile sensing in real–time. In fact, having the system two degrees of freedom, to follow a planned path for the whole system it will suffice that two state variables are made to follow their planned trajectory accurately enough. In our case, we try to control the coordinates of the contact point on the lower finger to track the trajectory resulting from planning, and use tactile feedback to make this control effective. The tracking controller is designed according to a standard P.D. + feedforward scheme. In fig.3 are reported the planned trajectories for the contact coordinates (dotted line), and the actual trajectories followed by the system are superimposed for comparison (solid line). A rather good tracking accuracy can be observed, which re-

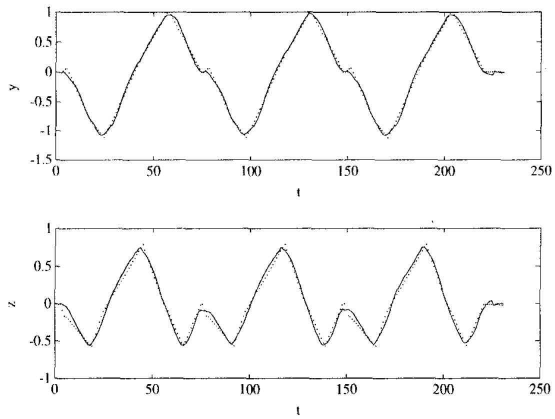

Figure 3: Planned and experimental trajectories of the contact point on the lower finger of the dexterous nonholonomic hand of fig.1

sulted in an overall accuracy in the reorientation maneuver of less than 1mm in position and a few degrees on the planned rotation.

6 Discussion

In order to prove the practicality of the proposed approach to the design of nonholonomic dextrous hands, more work has to be done under several respects. In particular, planning should be demonstrated for more general object shapes. At present, we are able to manipulate an object of arbitrary (regular) shape by using an adapted version of a continuation method proposed by Sussmann [18]. However, in its practical implementation this method, just like other related approximate iterative techniques (see e.g. Fernandes, Gurvits, and Li [6], and Divelbiss and Wen, [5]) suffers from an excessive demand of time for planning. More effective planners are being considered for objects belonging to classes of practical interest. Another important topic of research is concerned with manipulation of objects whose shape is not known a priori, and can be explored while manipulating through the use of tactile sensing.

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