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# GEOMETRIC CONTROL TECHNIQUES FOR MANIPULATION SYSTEMS

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Abstract: The control of robotic manipulation is investigated. Manipulation system analysis and control are approached in a general framework. The geometric aspect of manipulation system dynamics is strongly emphasized by using the well developed techniques of geometric multivariable control theory. The focus is on the control of the crucial outputs in robotic manipulation, namely the reachable internal forces and the rigid–body object motions. A state–feedback control procedure is outlined for decoupling these outputs and finally special attention is devoted to the synthesis of the state observer.

Keywords: Manipulators, Geometric Approach, Noninteraction, State Observer.

## 1. INTRODUCTION

The class of robotic systems this paper is focused on are referred to as general manipulation systems. These are mechanical structures more complex than conventional serial–linkage arms. The coordinated use of multiple fingers in a robot hand or, similarly, of multiple arms in cooperating tasks; the use of inner links of a robot arm (or finger) to hold an object, and the exploitation of parallel mechanical structures, are all examples of non–conventional usage of mechanisms for manipulation. Robotic hands can be considered as paradigms of general manipulation systems.

The presence of unilateral contact phenomena between different parts of the mechanical structure is a special feature of manipulation systems. Mechanical contacts between the robotic parts and the environment can be viewed as unactuated (passive) joints and, for this reason, they make manipulation system control quite involved.

The analysis of dynamics and the control of manipulation systems becomes more complex when it is not possible to control contact force in all directions. This usually happens when the number $\mathbf{q}$ of DoF's of the robotic device is smaller than $t$, the dimension of the contact force space. In (Prattichizzo and Bicchi, 1996), such a case is defined as "defective grasp".

The importance of defective grasps has been underlined, for the first time in "whole–hand" manipulation (Salisbury, 1987), where all links of the hand may be exploited to manipulate objects (see Fig. 1).

In industrial applications, kinematic defectivity is a common factor of almost all grippers used to grasp industrial parts. Consider, for instance, the simple mechanism in Fig. 2 of Section 6. It will be shown that it exhibits a defective grasp.

The main goal of dexterous manipulation tasks consists of controlling the motion of the manipulated object along with the grasping forces exerted on the object. In the robotics literature, the general problem of force/motion control is known as "hybrid control". For a broad overview on this topics, the reader is referred to (Murray *et al.*, 1994), (Siciliano, 1996) and the references therein.

In force/motion control, a very interesting aspect is the decoupling control. Roughly speaking, the multi–input, multi–output manipulation system is decoupled if each output vector, namely the grasping force and the object position vectors, can be *independently* controlled by corresponding set of generalized input forces. Such a structure is desirable in a considerable number of advanced applications, including micromanipulation of tissues in surgery and in laparoscopy or assembly and manipulation of non–rigid (rubber or plastic) parts in industry.

In all the examples above, it could be very dangerous to increase the squeezing force while giving rise to undesired, even if transient, object motions. Such a problem is common to all those hybrid controllers which do not rank noninteraction as a specific goal.

In this paper the noninteraction of contact forces and object motions is presented as a structural property of general manipulation systems. We prove that it is possible to decouple the object position and the squeezing force control for a wide class of manipulation systems by using a state–space feedback controller.

Moreover, the problem of synthesizing the state observer in the presence of unaccessible disturbing forces and torques acting on the objects is investigated.

The framework used in this paper is the geometric approach to the structural synthesis of multivariable systems. For a broad overview the reader is referred to (Basile and Marro, 1992), (Wonham, 1979) and references therein.

## 2. PRELIMINARIES

The manipulation system dynamics is linearized at an equilibrium configuration. The use of linearized model dynamics in the analysis of general manipulation systems is believed to be a significant advancement with respect to the literature, which is almost solely based on quasi–static models, especially for defective systems, and in fact provides richer results and better insights.

For a detailed discussion of dynamics and the derivation of the linearized model the reader is referred to previous works by the authors, (Bicchi and Prattichizzo, 1995) and (Prattichizzo and Bicchi, 1996).

Notation and some results on the linearized dynamics of general manipulation systems, are summarized in this section.

Let $\mathbf{q} \in \mathbb{R}^q$ be the vector of joint positions, $\tau \in \mathbb{R}^q$ the vector of joint forces and/or torques, $\mathbf{u} \in \mathbb{R}^d$ the vector locally describing the position and the orientation of a frame attached to the object and finally $\mathbf{w} \in \mathbb{R}^d$ the vector of external disturbances acting on the object. Let further introduce the vector $\mathbf{t} \in \mathbb{R}^t$ whose components include contact forces and torques.

Assume that contact forces arise from a lumped–parameter model of visco–elastic phenomena at the contacts, summarized by the stiffness matrix $\mathbf{K}$ and the damping matrix $\mathbf{B}$. The Jacobian $\mathbf{J}$ and the grasp matrix $\mathbf{G}$ are defined as usual as the linear maps relating the velocities of the contact points on the links and on the object, to the joint and object velocities, respectively.

Besides advanced robotic tasks discussed in the introduction, where visco–elastic contact model is mandatory, it might be worthwhile to mention another reason, discussed in (Prattichizzo, 1995), for taking into account the visco–elastic contact model. It was shown that if the grasp is hyperstatic, i.e. $\ker(\mathbf{J}^T) \cap \ker(\mathbf{G}) \neq \mathbf{0}$, the rigid–body contact model leaves the nonlinear dynamics undetermined and, consequently, the visco–elastic model of contact interaction becomes mandatory.

Consider a reference equilibrium configuration $(\mathbf{q}, \mathbf{u}, \dot{\mathbf{q}}, \dot{\mathbf{u}}, \tau, \mathbf{t}) = (\mathbf{q}_o, \mathbf{u}_o, \mathbf{0}, \mathbf{0}, \tau_o, \mathbf{t}_o)$, such that $\tau_o = \mathbf{J}^T \mathbf{t}_o$ and $\mathbf{w}_o = -\mathbf{G}\mathbf{t}_o$. In the neighbourhood of such an equilibrium, linearized dynamics of the manipulation system can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \tau' + \mathbf{B}_w \mathbf{w}', \qquad (1)$$

where state, input and disturbance vectors are defined as the departures from the reference equilibrium configuration:

$$\mathbf{x} = \left[(\mathbf{q}-\mathbf{q}_o)^T \ (\mathbf{u}-\mathbf{u}_o)^T \ \dot{\mathbf{q}}^T \ \dot{\mathbf{u}}^T\right]^T,$$

$$\tau' = \tau - \mathbf{J}^T\mathbf{t}_o, \ \mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o \ \text{ and}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \ \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \ \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix},$$

where $\mathbf{M}_h$ and $\mathbf{M}_o$ the inertia matrices of the manipulator and the object, respectively. To simplify notation we will henceforth omit the prime in $\tau'$ and $\mathbf{w}'$.

Neglecting rolling phenomena at the contacts, assuming a locally isotropic model of visco–elastic phenomena and assuming that local variations of the jacobian and grasp matrices are small, simple expressions are obtained for $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$ and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$, where $\mathbf{M} = \text{diag}(\mathbf{M}_h, \mathbf{M}_o)$, $\mathbf{P}_k = \mathbf{S}^T\mathbf{K}\mathbf{S}$, $\mathbf{P}_b = \mathbf{S}^T\mathbf{B}\mathbf{S}$, and $\mathbf{S} = [\mathbf{J} \ \ -\mathbf{G}^T]$.

## 3. CONTROLLED OUTPUTS

In this paper, it has been assumed that contact points do not change. The manipulation is studied in those interval of time when contact points hold whitout rolling and/or sliding. Thus, manipulation control goal involves mainly the control of grasp and the tracking of desired object trajectory.

As regards the first control requirement let us introduce the *internal* forces. Usually, forces belonging to the null space of grasp matrix $\mathbf{G}$ are referred to as "internal forces" which are contact forces with zero resultant on the object. Such forces enable the robotic device to grasp the object and play a fundamental role in controlling the manipulation task. A suitable control of internal forces allows the manipulation system to counteract the possible grasp failure caused by disturbance actions on the object. In (Prattichizzo and Bicchi, 1996) manipulation systems with $\ker(\mathbf{G}) \neq \{\mathbf{0}\}$ were defined as *graspable* systems.

As regards object trajectories, rigid–body kinematics play a particular function in manipulation control. Rigid–body kinematics have been studied in a quasi–static setting in (Bicchi *et al.*, 1995) and in terms of unobservable subspaces in (Bicchi and Prattichizzo, 1995). In both cases rigid kinematics were described by the base matrix $\mathbf{\Gamma}$ whose columns form a basis for $\ker\left[\mathbf{J}\ \ -\mathbf{G}^T\right] = \text{range}(\mathbf{\Gamma})$ where

$$\mathbf{\Gamma} = \left[\mathbf{\Gamma}_{qc}^T\ \mathbf{\Gamma}_{uc}^T\right]^T, \quad \text{and} \quad \mathbf{J}\mathbf{\Gamma}_{qc} = \mathbf{G}^T\mathbf{\Gamma}_{uc}. \quad (2)$$

Observe that, for the sake of brevity, it is assumed here that the system is not *redundant*: $\ker(\mathbf{J}) = \{\mathbf{0}\}$ and that it is not *indeterminate*: $\ker(\mathbf{G}^T) = \{\mathbf{0}\}$, see (Bicchi *et al.*, 1995) for further details.

The column space of $\mathbf{\Gamma}$ consists of coordinated rigid–body motions of the mechanism, for the manipulator ($\mathbf{\Gamma}_{qc}$) and the object ($\mathbf{\Gamma}_{uc}$) components. They do not involve visco–elastic deformations at contacts and can be regarded as low–energy motions. In this sense, they represent the easiest way to move the object.

In the following, a special subspace of internal forces and the rigid–body object motions are characterized as output matrices of the linearized dynamics, see Section 2. These outputs, namely $\mathbf{t}'$ and $\mathbf{u}'$ (henceforth $\mathbf{t}$ and $\mathbf{u}$), represent variations of contact force and object position vectors from the relative equilibrium values.

Before introducing the controlled outputs, let us recall the concept of *contact–kinematics defectivity*, or briefly *defectivity*. According to (Prattichizzo and Bicchi, 1996) and (Prattichizzo *et al.*, 1996*a*), a given grasp is called *contact–kinematics defective* if $\ker(\mathbf{J}^T) \neq \{\mathbf{0}\}$. As pointed out, the grasp

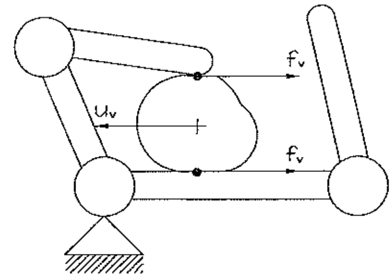


Fig. 1. Defective grasp: $\ker(\mathbf{J}^T) \neq \{0\}$. Contact force $\mathbf{f}_v$ and object position $\mathbf{u}_v$ are not controllable by joint torques.

*defectivity* deeply affects contact force and object motion controllability which, in general, is lost. Fig. 1 pictorially describes some uncontrollable directions of contact forces $\mathbf{f}_v$ and object motions $\mathbf{u}_v$ for a simple 3–DoF's defective device.

Recall that whenever the number of joints is lower than the number of elements of the contact force, as in the simple grippers of Fig. 1 and 2, it ensues that $\ker(\mathbf{J}^T) \neq \{\mathbf{0}\}$ and the grasp is defective.

Although, in the presence of defectivity, contact forces $\mathbf{t}$ and object motions $\mathbf{u}$ loose the output controllability, it was shown in (Prattichizzo and Bicchi, 1996) that the output controllability property holds for their projection on the subspace of *reachable internal* forces $\mathbf{t}_i$ and of *rigid–body object* motions $\mathbf{u}_c$. Moreover, if the output vector is chosen by grouping such projections $\mathbf{y} = \left(\mathbf{t}_i^T\ \mathbf{u}_c^T\right)^T$, not only $\mathbf{y}$ is consistent, i.e. output controllable, but it also exhausts the control capability by making square the input–output representation of dynamics.

The *reachable internal* contact forces $\mathbf{t}_i$ are defined as the projection of the force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$:

$$\begin{aligned} &\mathbf{t}_i = \mathbf{E}_{ti}\mathbf{x} \quad \text{where} \\ &\mathbf{E}_{ti} = (\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{Q}^T\left[\mathbf{Q}\ \mathbf{0}\ \mathbf{Q}\ \mathbf{0}\right]; \\ &\mathbf{Q} = (\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}, \end{aligned} \quad (3)$$

and the *rigid–body* object motions $\mathbf{u}_c$ are defined as the projection of the object displacement $\mathbf{u}$ onto the column space of $\mathbf{\Gamma}_{uc}$:

$$\begin{aligned} &\mathbf{u}_c = \mathbf{E}_{uc}\mathbf{x}; \quad \text{where} \\ &\mathbf{E}_{uc} = (\mathbf{\Gamma}_{uc}^T\mathbf{\Gamma}_{uc})^{-1}\mathbf{\Gamma}_{uc}^T\left[\mathbf{0}\ \mathbf{I}\ \mathbf{0}\ \mathbf{0}\right]. \end{aligned} \quad (4)$$

Notice that to simplify notation, matrices $(\mathbf{Q}^T\mathbf{Q})^{-1}$ and $(\mathbf{\Gamma}_{uc}^T\mathbf{\Gamma}_{uc})^{-1}$, will be omitted.

## 4. NONINTERACTING CONTROL

The following theorem, proven in (Prattichizzo *et al.*, 1996*a*), states that the force/motion decoupling problem is a structural property of general manipulation systems.

*Theorem 1.* **(Noninteraction)** Consider the linearized manipulation system of Section 2. If $\ker(\mathbf{G}^T) = \{\mathbf{0}\}$, there exists a stabilizing state–feedback control law, $\tau = \mathbf{F}\mathbf{x} + \tau^*$ and an input partition $\tau^* = \mathbf{U}_{ti}\mathbf{u}_{ti} + \mathbf{U}_{uc}\mathbf{u}_{uc}$ which decouples reachable internal forces $\mathbf{t}_i$ and rigid–body object motions $\mathbf{u}_c$.

*Remark 1.* Theorem 1 shows that a control law and a joint torques partition exists such that, for zero initial conditions, each input only affects the relative output.

The geometric concept from which the previous result develops is the *$\mathcal{S}$–constrained controllability*. It consists of those state space vectors reachable through trajectories entirely lying in the constraining subspace $\mathcal{S}$.

In other words, for the aforementioned outputs $\mathbf{t}_i$ and $\mathbf{u}_c$, there exists a decoupling and stabilizing state feedback matrix $\mathbf{F}$, along with two input partition matrices $\mathbf{U}_{ti}$ and $\mathbf{U}_{uc}$ such that, for the dynamic triples

$$\begin{aligned} &(\mathbf{E}_{ti},\ \mathbf{A}+\mathbf{B}_\tau\mathbf{F},\ \mathbf{B}_\tau\mathbf{U}_{ti})\,; \\ &(\mathbf{E}_{uc},\ \mathbf{A}+\mathbf{B}_\tau\mathbf{F},\ \mathbf{B}_\tau\mathbf{U}_{uc})\,, \end{aligned} \tag{5}$$

it holds:

$$\begin{aligned} \mathcal{R}_{ti} &= \min\ \mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F},\ \mathbf{B}_\tau\mathbf{U}_{ti}) \subseteq \ker(\mathbf{E}_{uc}); \\ \mathbf{E}_{ti}\mathcal{R}_{ti} &= \ \mathrm{range}(\mathbf{E}_{ti}); \\ \mathcal{R}_{uc} &= \min\ \mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F},\ \mathbf{B}_\tau\mathbf{U}_{uc}) \subseteq \ker(\mathbf{E}_{ti}); \\ \mathbf{E}_{uc}\mathcal{R}_{uc} &= \ \mathrm{range}(\mathbf{E}_{uc}). \end{aligned} \tag{6}$$

Here, $\min\ \mathcal{I}(\mathbf{A},\ \mathrm{range}(\mathbf{B})) = \sum_{i=0}^{n-1}\mathbf{A}^i\ \mathrm{range}(\mathbf{B})$ is the minimum $\mathbf{A}$–invariant subspace containing $\mathrm{range}(\mathbf{B})$ and $\max\mathcal{I}(\mathbf{A}, \ker(C)) = \bigcap_{i=0}^{n-1}\mathbf{A}^i\ker(C)$ is the maximum $\mathbf{A}$–invariant subspace contained in $\ker(\mathbf{C})$ with respect to the triple $(\mathbf{A}, \mathbf{B}, \mathbf{C})$.

Moreover, partition matrices $\mathbf{U}_{uc}$ and $\mathbf{U}_{ti}$ satisfy the following relationships

$$\begin{aligned} \mathrm{range}(\mathbf{B}_\tau\mathbf{U}_{uc}) &= \ \mathrm{range}(\mathbf{B}_\tau)\cap\mathcal{R}_{uc}; \\ \mathrm{range}(\mathbf{B}_\tau\mathbf{U}_{ti}) &= \ \mathrm{range}(\mathbf{B}_\tau)\cap\mathcal{R}_{ti} \end{aligned} \tag{7}$$

and the stabilizing matrix $\mathbf{F}$ is such that

$$\begin{aligned} (\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{uc} &\subseteq \mathcal{R}_{uc}; \\ (\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{ti} &\subseteq \mathcal{R}_{ti}. \end{aligned} \tag{8}$$

The decoupling controller is that sketched in Fig. 3.

## 5. STATE OBSERVER SYNTHESIS

While the measurement of joint positions can be easily obtained, in robotics manipulation the object posture is measurable only by using expensive sensor systems such as vision–based sensors, cf. (Hager and Hutchinson, 1987), or global positioning systems (GPS). Hence from a practical point of view, the state is not available and a state observer must be designed to implement the decoupling control law.

Assume that the sensed output $\mathbf{y}_s$ of the robotic manipulator consists of the joint positions and contact force vectors,

$$\begin{aligned} &\mathbf{y}_s = \begin{bmatrix}\mathbf{q}\\ \mathbf{t}\end{bmatrix} = \mathbf{C}\mathbf{x} \quad \text{with} \\ &\mathbf{C} = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}\mathbf{J} & -\mathbf{K}\mathbf{G}^T & \mathbf{B}\mathbf{J} & -\mathbf{B}\mathbf{G}^T\end{bmatrix}. \end{aligned} \tag{9}$$

In (Prattichizzo, 1995), it was shown that the linearized dynamics of general manipulation systems is detectable from $\mathbf{y}_s$, thus the identity observer can be considered for the asymptotic estimation of the system state.

It should be remarked that in order to realize the identity observer, not only the system detectability but also input accessibility is required. Unfortunately, the external object disturbance $\mathbf{w}$ of equation (1) is not accessible.

In this section the asymptotic estimation of state $\mathbf{x}$, in the presence of the unaccessible disturbance $\mathbf{w}$ is investigated.

It can be easily proven that, in the presence of unaccessible disturbances, the differential equation of the error $\mathbf{e}$ between the identity state observer output and the actual state is obtained as

$$\mathbf{e}(t) = (\mathbf{A}+\mathbf{L}\mathbf{C})\mathbf{e}(t) + \mathbf{B}_w\mathbf{w}(t).$$

The estimation of the state does not converge asymptotically to zero, even if $\mathbf{A}+\mathbf{L}\mathbf{C}$ is a stable matrix, but converges asymptotically to the subspace $\min\ \mathcal{I}(\mathbf{A}+\mathbf{L}\mathbf{C}, \mathbf{B}_w)$, the reachable subspace of the error dynamics.

According to (Basile and Marro, 1992), in order to maximize the dimension [1] of $(\min\ \mathcal{I}(\mathbf{A}+\mathbf{L}\mathbf{C}, \mathbf{B}_w))^\perp$, it is convenient to choose $\mathbf{L}$ such that $\min\ \mathcal{I}(\mathbf{A}+\mathbf{L}\mathbf{C}, \mathbf{B}_w)$ has the smallest dimension.

The following proposition, proven in (Prattichizzo *et al.*, 1996*b*), solves the optimization problem above for the linearized dynamics of general manipulation systems.

[1] The dimension of subspaces where the state can be estimated with zero asymptotic error.

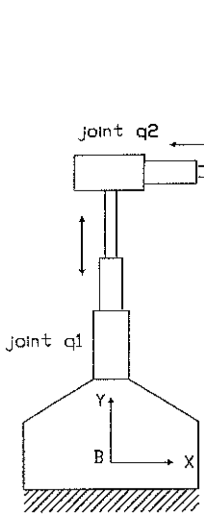


Fig. 2. Planar 3–DoF's cartesian manipulator. It exhibits a defective ($\ker(\mathbf{J}^T) = \{\mathbf{0}\}$) grasp.

*Proposition 1.* Consider the linearized dynamics of eq.(1). There always exists a stabilizing matrix $\mathbf{L}$ such that

$$\min \, \mathcal{I}(\mathbf{A} + \mathbf{LC}, \mathbf{B}_w) = \, \mathrm{range}\,(\mathbf{B}_w),$$

Moreover matrix $\mathbf{L}$ stabilizes $\mathbf{A} + \mathbf{LC}$ and is such that

$$(\mathbf{A} + \mathbf{LC}) \, \mathrm{range}\,(\mathbf{B}_w) \subseteq \, \mathrm{range}\,(\mathbf{B}_w).$$

The proposition can be easily proven

- by recalling that minimizing the subspace dimension over the stabilizing matrices $\mathbf{L}$ is equivalent to choosing a stabilizing matrix $\mathbf{L}$ which transforms the minimal conditioned invariant in $(\mathbf{A}, \ker(\mathbf{C}))$ containing $\mathbf{B}_w$ into an $(\mathbf{A} + \mathbf{LC})$–invariant and
- by observing the $\mathrm{range}\,(\mathbf{B}_w)$ is conditioned invariant in $(\mathbf{A}, \ker(\mathbf{C}))$.

## 6. CASE STUDY

In this section numerical results are reported for the simple defective gripper described in Fig. 2. It is a planar 3–DoF's cartesian manipulator and has been chosen in order to show the effectiveness of previous results for industrial grippers.

In the base frame B, the contact *centroids*, cf. (Bicchi *et al.*, 1995), are $c_1 = (2, 2)$, $\mathbf{c}_2 = (2, 3)$ and the object center of mass is $c_b = (2, 2.5)$ while the transpose of the Jacobian and the grasp matrix assume the following values

$$\mathbf{J}^T = \begin{bmatrix} 0\;1\;0\;0 \\ 1\;0\;1\;0 \\ 0\;0\;0\;1 \end{bmatrix}; \;\; \mathbf{G} = \begin{bmatrix} 1\;0 & \;1\;0 \\ 0\;1 & \;0\;1 \\ 0.5\;0 & -0.5\;0 \end{bmatrix}.$$

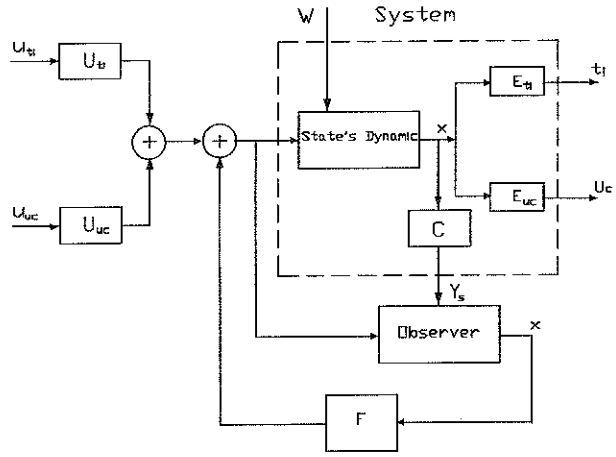


Fig. 3. Force/motion decoupling controller.

The inertia matrices of the object and manipulator along with stiffness and damping matrices at the contacts are assumed to be normalized to the identity matrix.

The controlled outputs are (a) the projection $\mathbf{t}_i$ of the contact forces on the 1–dimensional subspace of reachable contact force $\mathrm{range}\,([0\,1\,0\,-1]^T)$ and (b) the projection of the rigid–body motion on the 2–dimensional subspace of object motions $\mathrm{range}\begin{bmatrix} 1\;0 \\ 0\;1 \\ 0\;0 \end{bmatrix}$ which, since $\mathbf{u} = \begin{bmatrix} \delta\mathbf{x} \\ \delta\mathbf{y} \\ \delta\theta \end{bmatrix}$, corresponds to translations of the object.

The decoupling controller is described in Fig. 3 and has been synthesized, according to Section 4, eq. 5, 7 and 8. The state–feedback matrix $\mathbf{F}$ and the input partition matrices $\mathbf{U}_{ti}$ and $\mathbf{U}_{uc}$ are obtained respectively as

$$\begin{bmatrix} -7 & 6.5 & -6 & -1 & -41 & 0 & -7.5 & -0.02 & -5.5 & -3 & -22 & 0 \\ 10 & -120 & 10 & -72 & 5 & 0 & 0.29 & -16 & 0.29 & 7.2 & -6.2 & 0 \\ -6.1 & 6.5 & -7.1 & -0.97 & -41 & 0 & -5.5 & -0.021 & -7.5 & -3.1 & -22 & 0 \end{bmatrix};$$

$$\begin{bmatrix} -0.707 \\ 0 \\ 0.707 \end{bmatrix}; \qquad \begin{bmatrix} 0 & -0.707 \\ 1 & 0 \\ 0 & -0.707 \end{bmatrix}.$$

Simulation results of Fig. 4 show the noninteracting property stated in Theorem 1. In column a) and b) contact forces $\mathbf{t}_i$ and object motions projections on the $\mathbf{x}$ and $\mathbf{y}$ directions are reported for simulations with zero disturbances. Column a) refers to a simulation where the unitary step is assigned to input $\mathbf{u}_{ti}$ and the zero value is assigned to both components of input $\mathbf{u}_{uc}$, see the block diagram of Fig 3. Column b) refers to a simulation where input $\mathbf{u}_{ti}$ is set to zero and a unitary step is assigned to both components of input $\mathbf{u}_{uc}$.

Results in Fig. 5 show the effects of the optimum state observer synthesis, where optimality is meant in the sense of Proposition 1. A disturbing force, whose projections on $\mathbf{x}$ and $\mathbf{y}$ are unitary, acts on the object, see Fig. 2. Controlled outputs are set to zero. Column a) shows the output behaviour with the feedback observer matrix $\mathbf{L}$ chosen according

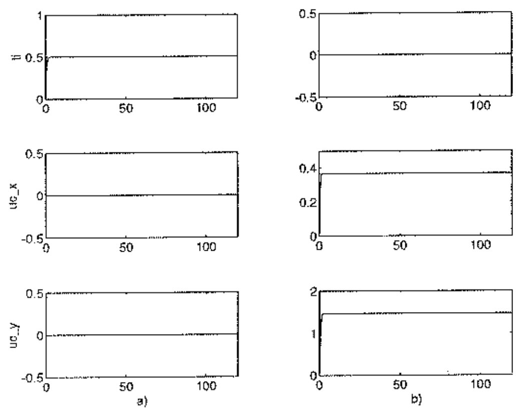


Fig. 4. Force/motion noninteraction. Column a) [b)] reports simulation results obtained with only internal force [object motions] input.

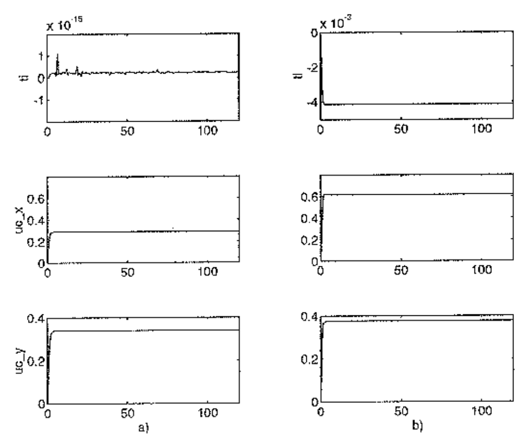


Fig. 5. Only the disturbing action: $\mathbf{w} = [1\ 1\ 0]^T$ acts on the system. Column a) shows the output behaviour when $\mathbf{L}$ is optimum. Column b) shows the output behaviour when $\mathbf{L}$ is chosen by using a simple pole pole placement procedure.

to Proposition 1. For results of column b), matrix $\mathbf{L}$ is synthesized by a simple pole placement procedure. Observe that the disturbance attenuation on the controlled output is more effective when the observer matrix $\mathbf{L}$ is optimum in the sense of Proposition 1.

## 7. CONCLUSIONS

The decoupling procedure discussed in this paper applies to robotic manipulation systems whose dynamics can be modelled according to Section 2. The class of manipulation system under investigation is wide enough to include a considerable number of grasp configurations, such as those using internal and/or extremal links to grasp objects, those with contact kinematic redundancy and so forth.

Due to the possible presence of defectivity, the control outputs were suitably chosen as the reachable internal forces and the rigid–body object motions.

The main results of this paper are summarized in Theorem 1 and Proposition 1. The first states that the force/motion noninteraction is a structural property of general manipulation systems. The second focuses on the estimation of the state in the presence of unaccessible disturbing forces acting on the object.

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