ADAPTIVE NON LINEAR CONTROL OF DYNAMIC MOBILE ROBOTS WITH PARAMETERS UNCERTAINTY[1]

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Contents

Abstract

Research of a modular stabilizing control law for uncertain, nonholonomic mobile systems with actuators limitation has been investigated. Modular design allows the definition of a stabilizing control law for the kinematic model. The presence of uncertainties in the actuators parameters or in the vehicle dynamics has been treated both adding suitable components to the Lyapunov function and using parameters adaptation laws (e.g. adaptive control and backstepping techniques). Simulations are reported for the set point stabilization of a unicycle like vehicle showing the feasibility of the proposed approach. Torque limitations for a unicycle like vehicle has been investigated using backstepping techniques for the vehicle tracking problem. Simulations are reported.

1 Introduction

Practical control of robots involves both the design of state feedback control laws and the modelling of the physical robotic platform used. Notwithstanding the efforts spent to identify the parameters of the mechanical system, control law robustness is more often increased using adaptive control (Jiang *et al.*, 2004). On the other hand, mechanical systems suffer also of limits on actuation that should be taken into account to prevent instability. Combining parameters uncertainties with actuator limits carries to a challenging yet open problem, particularly if the system to control is nonholonomic.

The problem of actuators limitation for velocity control of kinematic nonholonomic systems has been solved for example in (Sontag and Malisoff, 1999) and (Jiang *et al.*, 2002) defining universal formulas for asymptotic stability, in (Nijmeijer *et al.*, 2001), with time varying control Lyapunov functions, or in (Beard and Ren, 2004) for air vehicles' control. In the present paper, the problem of adaptive nonlinear control for generic kinematic nonholonomic system in the presence of actuators limit is considered assuming a linear relation between the control space and the uncertainties without knowledge on the parameter values.

In literature, the problem of stabilizing a unicycle like vehicle (i.e. driftless nonholonmic system) with both uncertain actuators and dynamic parameters has been solved for regulation (Aguiar *et al.*, 2000) and for path–following (Soetano *et al.*, 2003) using adaptive nonlinear and backstepping control assuming the knowledge of the uncertainty sign. In (Tso *et al.*, 2000) the tracking control of uncertain dynamic nonholonomic system has been solved for systems transformable in *Extended One–Generator Multi–Chain Form*, defining universal formulas for Lyapunov stabilization.

To the best of authors' knowledge, while the tracking problem has been solved considering the presence of maximum velocity constraints, very little has been done in the investigation of tracking control laws with limited torques.

This paper presents an attempt to solve the control of nonholonomic mobile robots using adaptive and switching control in presence of uncertainties related to actuation and dynamic, coping with limitations on actuators saturation. The underlying idea is that each component of the final control law can be modularly composed using Lyapunov functions, starting from a stabilizing controller thought for the kinematic, perfectly known model. The problem of torque limitations for the unicycle tracking problem has been solved using switching techniques. Simulations are reported demonstrating the feasibility of the proposed approach.

2 Adaptive nonlinear control

Let us consider a generic, nonholonomic, input–affine nonlinear system

$$\dot{q} = f(q) + G(q)u \tag{1}$$

where $q \in \mathbb{R}^n$ is the state space vector, $f(q)$ and $G(q)$ are the drift and the input vector fields respectively and $u \in \mathbb{R}^m$ are the available controls. Let $u(q) \in U \subset \mathbb{R}^m$ be the control law obtained for the kinematic system and satisfying the control task. Indeed, it exists a positive definite Lyapunov function $\mathbf{V}_1(q) > 0$, with

$$\dot{\mathbf{V}}_1(q) = \nabla \mathbf{V}_1(q) \left[f(q) + G(q)u(q) \right] < 0 \tag{2}$$

whenever $q \neq 0$. The system is then asymptotically stable using the stabilizing control law $u(q)$.

Consider now a new input space $V \subset \mathbb{R}^m$ and the isomorphism $\tilde{F}^{-1}$: $U \to V$. The control input $u_\nu \in V$ may be, for instance, the actual, low level velocity vector available on the physical nonholonomic system, while the control input u can be viewed as a control abstraction, e.g. the steering velocities of the kinematic system model. The stabilizing controller will be trivially $u_\nu = \tilde{F}^{-1}(u)$. Let the isomorphism be a bilinear w.r.t. U and some parameters $\eta \in \mathbb{R}^p$, then:

$$u_\nu = \tilde{F}^{-1}(u, \eta) = F^{-1}(u)\Theta_\eta \bar{\mathbf{1}}$$

where, imposing $p = m$, $\bar{\mathbf{1}} = [1, 1, \ldots, 1]^T \in \mathbb{R}^p$ and $\Theta_\eta = \operatorname{diag}(\eta_i)$. From the hypothesis of the isomorphism $\tilde{F}^{-1}$, it is possible to define the invertible mapping function and the relative Lyapunov function:

$$\begin{aligned} &u = \tilde{F}(u_\nu, \eta) = F(u_\nu)\Theta_\eta^{-1}\bar{\mathbf{1}} \\ &\dot{\mathbf{V}}_1(q, \eta) = \nabla \mathbf{V}_1(q) \left[f(q) + G(q)F(u_\nu)\Theta_\eta^{-1}\bar{\mathbf{1}} \right] < 0 \end{aligned}$$

Considering an imperfect knowledge of the parameters η, $\dot{\mathbf{V}}_1(q, \eta)$ is not defined any further. Defining $\tilde{\eta} = \hat{\eta} - \eta$ as the parameters error on the estimations $\hat{\eta}$, we have:

$$\begin{aligned} &\hat{u}_\nu(q) = \tilde{F}^{-1}(u, \hat{\eta}) = F^{-1}(u)\hat{\Theta}_\eta \bar{\mathbf{1}} \\ &\hat{u}(q) = \tilde{F}(\hat{u}_\nu, \eta) = F(\hat{u}_\nu)\Theta_\eta^{-1}\bar{\mathbf{1}} \\ &u(q) = \tilde{F}(u_\nu, \eta) = \tilde{F}(\hat{u}_\nu, \hat{\eta}) = F(\hat{u}_\nu)\hat{\Theta}_\eta^{-1}\bar{\mathbf{1}} \end{aligned}$$

where $u(q)$ is the desired, kinematically stabilizing control law $\hat{u}(q) \in U$ and $\hat{u}_\nu(q) \in V$ are the actual control input and the low level control input vectors respectively, affected by the parameter estimation uncertainties $\hat{\eta}$. The control $\hat{u}(q)$ is applied to the system that has the true parameter η. Using the control Lyapunov function $\mathbf{V}(q) = \mathbf{V}_1(q)$, the time derivative becomes:

$$\begin{aligned}\dot{\mathbf{V}}(q,\eta,\tilde{\eta}) &= \nabla\mathbf{V}_1(q)f(q) + \nabla\mathbf{V}_1(q)G(q)F(\hat{u}_\nu)\Theta_\eta^{-1}\bar{\mathbf{1}} \\ &= \nabla\mathbf{V}_1(q)f(q) + \nabla\mathbf{V}_1(q)G(q)U_q\hat{\Theta}_\eta\Theta_\eta^{-1}\bar{\mathbf{1}} \\ &= \nabla\mathbf{V}_1(q)f(q) + \\ &\quad \nabla\mathbf{V}_1(q)G(q)U_q\left(\tilde{\Theta}_\eta + \Theta_\eta\right)\Theta_\eta^{-1}\bar{\mathbf{1}} \\ &= \dot{\mathbf{V}}_1(q) + \nabla\mathbf{V}_1(q)G(q)U_q\tilde{\Theta}_\eta\Theta_\eta^{-1}\bar{\mathbf{1}}\end{aligned}$$

where $U_q = \text{diag}(u_i(q))$.

Under the assumption that the parameters are unknown but constant, i.e. $\dot{\tilde{\eta}} = \dot{\hat{\eta}}$, consider $\mathbf{V}(q,\eta,\tilde{\eta}) = \mathbf{V}_1(q) + \mathbf{V}_\eta(\eta,\tilde{\eta})$, where:

$$\begin{cases}\mathbf{V}_\eta(\eta,\tilde{\eta}) = \frac{1}{2}\bar{\mathbf{1}}^T\tilde{\Theta}_\eta^T\Theta_\eta^{-T}\Gamma\tilde{\Theta}_\eta\bar{\mathbf{1}} > 0 \\ \dot{\mathbf{V}}_\eta(\eta,\tilde{\eta}) = \bar{\mathbf{1}}^T\tilde{\Theta}_\eta^T\Theta_\eta^{-T}\Gamma\dot{\tilde{\Theta}}_\eta\bar{\mathbf{1}}\end{cases} \tag{3}$$

where $\Gamma > 0$ and symmetric.

Choosing the adaptation of the uncertain parameters as:

$$\dot{\tilde{\Theta}}_\eta\bar{\mathbf{1}} = \dot{\tilde{\eta}} = \dot{\hat{\eta}} = -\Gamma^{-1}U_q^T G(q)^T\nabla\mathbf{V}_1(q)^T, \tag{4}$$

the time derivative of the Lyapunov function (3) becomes [1]:

$$\dot{\mathbf{V}}_\eta(q,\eta,\tilde{\eta}) = -\nabla\mathbf{V}_1(q)G(q)U_q\tilde{\Theta}_\eta\Theta_\eta^{-1}\bar{\mathbf{1}}$$

that ensures the perfect compensation of the undefined sign term. Hence, the final control Lyapunov function and its time derivative are:

$$\begin{cases}\mathbf{V}(q,\eta,\tilde{\eta}) = \mathbf{V}_1(q) + \mathbf{V}_\eta(\eta,\tilde{\eta}) > 0 \\ \dot{\mathbf{V}}(q,\eta,\tilde{\eta}) = \dot{\mathbf{V}}_1(q) < 0\end{cases},$$

and the system with uncertain parameters inherits the stability features (simple or asymptotic) of the kinematic system without uncertain parameters since $\dot{\mathbf{V}}(q,\eta,\tilde{\eta})$ is negative semidefinite with respect to the whole state space $(q,\tilde{\eta}) = (0,\tilde{\eta})$. The uncertain parameter estimation does not necessarily converges to zero, as is usual in the adaptive control framework, while the system correctly does its job (this can be proved using Lasalle's theorem (Hahn, 1963)).

The steps to follow to apply the proposed control law are described hereinafter:

[1] where we use the fact that the diagonal matrices $\tilde{\Theta}_\eta$ and Θ_η^{-1} commute.

1. Compute the desired kinematic control law $u(q)$;
2. Project the control law in the new input space $\hat{u}_\nu(q) = \tilde{F}^{-1}(u(q), \hat{\eta})$;
3. From $u(q)$, the adaptation parameter law is computed (see equation (4));
4. Apply the computed low level controls $\hat{u}_\nu(q)$ to the vehicle.

This procedure emphasizes the modular design of the proposed technique. It is worthwhile to note that for driftless nonholonomic systems with uncertain parameters of the form (1), the adaptation law can be avoided if the sign of the uncertain parameter is known (see (Soetano *et al.*, 2003) and (Aguiar *et al.*, 2000)).

2.1 Actuators' limits

Let us consider $\eta_i > 0$, with $i = 1, \ldots, m$ and $\dot{\eta}_i = 0$, and a limited input velocity $|\hat{u}_i| \leq u_{max_i}$, with $u_{max_i} > 0, \forall i = 1, \ldots, m$. The velocity constraint is satisfied if

$$|\hat{u}(q)| = |\tilde{u}(q) + u(q)| \leq |\tilde{u}(q)| + |u(q)| \leq u_{max} \, .$$

Due to the presence of uncertainties, the velocity error in the low level inputs $\tilde{u}_\nu(q) = \hat{u}_\nu(q) - u_\nu(q)$ can be rewritten as

$$\tilde{u}_\nu = F^{-1}(u(q))\tilde{\Theta}_\eta \bar{\mathbf{1}} \tag{5}$$

that ensures the separation of each uncertainty with respect to the control input space (for the linearity of the input transformation inverse $\tilde{F}^{-1}$). Applying the linear operator F to (5) and multiplying both side by $\Theta_\eta^{-1}\bar{\mathbf{1}}$, one gets:

$$\tilde{u} = F(\tilde{u}_\nu(q))\Theta_\eta^{-1}\bar{\mathbf{1}} = U_q \tilde{\Theta}_\eta \Theta_\eta^{-1}\bar{\mathbf{1}} \tag{6}$$

For simplicity's sake, let us now examine each single uncertainty separately, since the parameter matrices Θ are of diagonal form. Consider

$$|\tilde{u}_i(q)| = |u_i(q))| \left| \frac{\tilde{\eta}_i}{\eta_i} \right| \, .$$

The limited velocity constraint affects the desired control inputs $u_i(q)$, with $i = 1, \ldots, m$, and the uncertainty parameters error and true value:

$$|\hat{u}_i(q)| \leq |\tilde{u}_i(q)| + |u_i(q)| = |u_i(q)| \left(\left| \frac{\tilde{\eta}_i}{\eta_i} \right| + 1 \right) \, .$$

Unfortunately, no assumptions can be made on the values of the estimation parameters along the controlled trajectories of the system. However, if

$$\left|\frac{\tilde{\eta}_i}{\eta_i}\right| \leq 1,\ \forall i = 1, \ldots, m \tag{7}$$

holds for each time $t > t_0$, where t_0 is the starting time, the limited velocity constraint is imposed directly on the desired, perfectly known, kinematic control $u(q)$, since

$$\begin{array}{l} |\hat{u}_i(q)| \leq |\tilde{u}_i(q)| + |u_i(q)| \leq 2|u_i(q)| \Rightarrow \\ |u_i(q)| \leq \frac{1}{2} u_{max_i},\ \forall i = 1, \ldots, m \end{array}$$

Since $\eta_i > 0$, by letting the uncertain parameters $0 < \hat{\eta}_i < 2\eta_i$, the condition (7) is fulfilled in the initial configuration (i.e. at t_0). Nevertheless, the constraints on the estimated parameters have to be satisfied on all the possible trajectories of the nonholonomic system. Choosing the Lyapunov function (3) and the adaptation parameters law (4), the nonholonomic system is controlled with limited controls by properly tuning the estimation parameter kinematic weighting matrix Γ. Moreover, if some measure of the controls applied to the robot are available, the adaptation parameter law can be further improved adding to equation (4) an error proportional term:

$$\dot{\hat{\eta}} = -\Gamma^{-1} U_q^T (G(q)^T \nabla \mathbf{V}_1(q)^T + \hat{u}(q) - u(q)),$$

in order to obtain a final control Lyapunov function:

$$\left\{ \begin{array}{l} \mathbf{V}(q, \eta\tilde{\eta}) = \mathbf{V}_1(q) + \mathbf{V}_\eta(\eta, \tilde{\eta}) > 0 \\ \dot{\mathbf{V}}(q, \eta, \tilde{\eta}) = \dot{\mathbf{V}}_1(q) - (\hat{u}(q) - u(q))^T U_q \Theta_\eta^{-1} \tilde{\Theta}_\eta \bar{\mathbf{1}} \end{array} \right. ,$$

that is n.d. as long as $\hat{\eta} > 0$. This is more evident if we replace $\hat{u}(q) - u(q)$ with (6) and use the commutativity between Θ_η^{-1} and $\tilde{\Theta}_\eta$:

$$\dot{\mathbf{V}}(q, \eta, \tilde{\eta}) = \dot{\mathbf{V}}_1(q) - \bar{\mathbf{1}}^T \tilde{\Theta}_\eta^T \Theta_\eta^{-T} U_q^T U_q \Theta_\eta^{-1} \tilde{\Theta}_\eta \bar{\mathbf{1}}$$

In such a way, the parameter estimation asimptotically converges to the true parameter value η if the system trajectories are "persistently exciting", i.e. if $\tilde{\eta} \to 0$ faster than $u(q) \to 0$. Therefore

$$\left|\frac{\tilde{\eta}_i}{\eta_i}\right| < 1,\ \forall t > t_0$$

and the nonholonomic system is driven in the final configuration with unknown parameters and with limited velocities.

It is worthwhile to note that $\tilde{\eta} \to 0$ is a steady state condition (i.e. $t \to +\infty$) that does not ensure $\dot{\tilde{\eta}} > 0$ locally (i.e. for $t_0 < t < \bar{t}$). Hence, the condition $\left|\frac{\tilde{\eta}_i}{\eta_i}\right| < 1$ would not hold anymore. Facing this problem is possible once the adaptation parameter law is modified as

$$\dot{\hat{\eta}} = -\Gamma^{-1} U_q^T (G(q)^T \nabla \mathbf{V}_1(q)^T + Q(\hat{u}(q) - u(q))),$$

where $Q > 0$ is a symmetric matrix obtained as an upper bound of the system input dynamic $G(q)^T \nabla \mathbf{V}_1(q)^T$.

3 Adaptive control for dynamic uncertain parameters

Let us consider a general mechanical system with nonholonomic constraints:

$$\begin{array}{l} B^\star(q)\ddot{q} + C^\star(\dot{q}, q)\dot{q} + G_u^\star(q) + A(q)^T \lambda = W^\star(q)\tau \\ A(q)\dot{q} = 0 \Rightarrow \dot{A}(q)\dot{q} + A(q)\ddot{q} = 0 \end{array} \quad (8)$$

Using standard manipulations of constrained dynamic systems (see for example (Tso *et al.*, 2000)), it is possible to decompose the system into two parts: the kinematic model and the relative dynamic model. Hence, let us consider a generic dynamic nonholonomic system, with drift term:

$$\left\{ \begin{array}{lcl} \dot{q} & = & f(q) + G(q)u \\ \dot{u} & = & B(q,\eta)^{-1}(W(q)\tau - C(\dot{q}, q, \eta)u - G_u(q,\eta) + \\ & & +\gamma^\star(q,\eta)) \end{array} \right. \quad (9)$$

where $q \in \mathbb{R}^n$ is the state vector of the kinematic model state space (i.e. generalized system variables), $f(q)$ and $G(q)$ are the system vector fields and $u \in \mathbb{R}^m$ are the kinematic controls. $\eta \in \mathbb{R}^p$ are the dynamic parameters of the mechanical nonholonomic system. Furthermore, $\gamma^\star(q,\eta)$ is a generic non linear term that is supposed to be linear with respect to the dynamic parameter $\gamma^\star(q,\eta) = \gamma(q)\eta$, that could appear from changes of coordinates. It is straightforward that:

$$B(q,\eta)\dot{u} + C(\dot{q}, q, \eta)u + G_u(q,\eta) = Y(\dot{u}, u, \dot{q}, q)\eta$$

where $Y(\dot{u}, u, \dot{q}, q)$ is the well known matrix regressor. Suppose that a stabilizing control law u for the kinematic system exists (e.g. the controller obtained in (Murrieri *et al.*, 2003) for a unicycle like vehicle). Hence, there exists a positive definite Lyapunov function $\mathbf{V}_1(q)$ whose time derivative satisfies (2). The same kinematic control law can be used also with the full dynamic system as the virtual control of a backstepping problem. Moreover, if the dynamic parameters are only partially known, the control law can be made robust w.r.t. parameter uncertainties by means of an adaptive framework. Define $\tilde{u}(q) = u_\tau(q) - u(q)$ as the control error and $\Delta = (\nabla \mathbf{V}_1(q) G(q))^T$, where u_τ is the dynamic system variable, i.e. the velocity control law of the kinematic subsystem. Using backstepping techniques, the asymptotic value of the control error can be driven to zero. Consider the torque control law:

$$\begin{aligned} \tau &= W(q)^{-1}(B(q,\eta)\dot{u}(q) + C(\dot{q},q,\eta)u(q)+ \\ &\quad +G_u(q,\eta) - \gamma^\star(q,\eta) - K_b\tilde{u} - \Delta) \end{aligned} \tag{10}$$

with K_b a square, positive definite matrix (the backstepping gain) that gives:

$$B(q,\eta)\dot{\tilde{u}} = -K_b\tilde{u} - \Delta - C(\dot{q},q,\eta)\tilde{u}$$

To prove that the proposed control law effectively stabilizes the system, we use the following control Lyapunov function:

$$\mathbf{V}_2(q,\tilde{u}) = \mathbf{V}_1(q) + \frac{1}{2}\tilde{u}^T B(q,\eta)\tilde{u}, \tag{11}$$

having time derivative:

$$\begin{aligned} \dot{\mathbf{V}}_2(q,\tilde{u}) &= \nabla \mathbf{V}_1(q) f(q) + \Delta^T u_\tau + \tilde{u}^T B(q,\eta)\dot{\tilde{u}}+ \\ &\quad +\tfrac{1}{2}\tilde{u}^T \dot{B}(q,\eta)\tilde{u} \\ &= \nabla \mathbf{V}_1(q) f(q) + \Delta^T u - \tilde{u}^T K_b \tilde{u} \\ &\quad +\tfrac{1}{2}\tilde{u}^T \left(\dot{B}(q,\eta) - 2C(\dot{q},q,\eta)\right)\tilde{u} \\ &= \dot{\mathbf{V}}_1(q) - \tilde{u}^T K_b \tilde{u} \end{aligned} \tag{12}$$

that is clearly negative definite. It is worth noting that the term added in (11) represents the kinetic energy of the vehicle and that the relation $\dot{B}(q,\eta) - 2C(\dot{q},q,\eta) = 0$ represents the Hamilton's principle on the energy conservation. The first term of the right side of (12) ensures the stability of the system while the second term ensures the convergence of $\tilde{u} \to 0$. As the backstepping gain matrix K_b increases, the latter convergence velocity increase as it is increasing the control effort as well.

For ease of notation, in what follows, we will suppress the explicit dependence of system matrices and controls by $q, \dot{q}, u, \dot{u}$. Consider now a partial knowledge of the dynamic parameters $\hat{\eta}$. Let $\tilde{\eta} = \hat{\eta} - \eta$ be the parameter estimation error and with $\tilde{M}(\tilde{\eta}) = \hat{M}(\hat{\eta}) - M(\eta)$ the estimation error on the generic system matrix M due to parameter uncertainties.

The torque control law is then:

$$\begin{array}{rcl} \tau & = & W^{-1}(\hat{B}(\hat{\eta})\dot{u} + \hat{C}(\hat{\eta})u + \hat{G}_u(\hat{\eta}) + \\ & & -\hat{\gamma}^{\star}(\hat{\eta}) - K_b\tilde{u} - \Delta), \end{array} \tag{13}$$

that replaced in $B(\eta)\dot{\tilde{u}}$ gives:

$$B(\eta)\dot{\tilde{u}} = -K_b\tilde{u} - \Delta + Y\tilde{\eta} - \gamma\tilde{\eta} - C(\eta)\tilde{u} \tag{14}$$

(recall that $\gamma^{\star}$, $\tilde{\gamma}^{\star}$, $\hat{\gamma}^{\star}$ are linear w.r.t. η, $\tilde{\eta}$ and $\hat{\eta}$ respectively).

Let $\dot{\eta} = 0$, i.e. constant unknown dynamic parameters, and the adaptation law of the parameters:

$$\dot{\tilde{\eta}} = \dot{\hat{\eta}} = -\Gamma^{-1}(Y^T - \gamma^T)\tilde{u} \tag{15}$$

and consider the composite Lyapunov function, with its time derivative:

$$\left\{ \begin{array}{l} \mathbf{V}_3(q, \tilde{u}, \tilde{\eta}) = \mathbf{V}_2(q, \tilde{u}) + \frac{1}{2}\tilde{\eta}^T\Gamma\tilde{\eta} \\ \begin{array}{rcl} \dot{\mathbf{V}}_3(q, \tilde{u}, \tilde{\eta}) & = & \nabla\mathbf{V}_1(q)f + \Delta^T u_\tau + \tilde{u}^T B(\eta)\dot{\tilde{u}} + \\ & & +\frac{1}{2}\tilde{u}^T\dot{B}(\eta)\tilde{u} + \tilde{\eta}^T\Gamma\dot{\tilde{\eta}} \end{array} \end{array} \right. \tag{16}$$

where $\Gamma > 0$ and symmetric.

Replacing (14) and (15) in (16), we obtain:

$$\begin{array}{rcl} \dot{\mathbf{V}}_3(q, \tilde{u}, \tilde{\eta}) & = & \nabla\mathbf{V}_1(q)f + \Delta^T u - \tilde{u}^T K_b\tilde{u} \\ & = & \dot{\mathbf{V}}_2(q, \tilde{u}) \end{array} \tag{17}$$

that is, once again, negative semidefinite if the desired kinematic control law makes $\mathbf{V}_1(q)$ negative definite. Therefore, the native control law $u(q)$ is used to stabilize the nonholonomic system, the kinematic control error $\tilde{u} \to 0$; the parameter estimation $\hat{\eta}$ does not converge necessarily to η, but still allows for the control task to be solved.

4 Adaptive control for dynamic and actuator's uncertain parameters

Consider again the mechanical system (8) and the stabilizing law (10). Let $\tau_\nu \in \mathbb{R}^m$ be a different set of torque inputs, related to some actuators' parameters $\eta_a \in \mathbb{R}^p$, whose generic non linear relation is $\tau = \tilde{F}_\tau(\tau_\nu, \eta_a)$. The full nonholonomic dynamics become:

$$\left\{ \begin{array}{rcl} \dot{q} & = & f + Gu \\ \dot{u} & = & B(\eta)^{-1}(W\tilde{F}_\tau(\tau_\nu, \eta_a) - C(\eta_d)u - G_u(\eta_d) + \\ & & +\gamma^{\star}(\eta_d)) \end{array} \right.$$

where η_d are dynamic parameters. The stabilizing control law for the new set of inputs is clearly: $\tau_\nu = \tilde{F}_\tau^{-1}(\tau, \eta_a)$. Defining $\tilde{\eta}_a = \hat{\eta}_a - \eta_a$ as the parameters error, function of the parameter estimation $\hat{\eta}_a$, and supposing that the new input field $\tilde{F}_\tau$ is bilinear w.r.t. τ and η_a, it is possible to assert that:

$$\begin{array}{l} \hat{\tau}_\nu = \tilde{F}_\tau^{-1}(\tau, \hat{\eta}_a) = F_\tau^{-1}(\tau)\hat{\Theta}_{\eta_a}\bar{\mathbf{1}} \\ \hat{\tau} = \tilde{F}_\tau(\hat{\tau}_\nu, \eta_a) = F_\tau(\hat{\tau}_\nu)\Theta_{\eta_a}^{-1}\bar{\mathbf{1}} \\ \tau = \tilde{F}_\tau(\tau_\nu, \eta_a) = \tilde{F}_\tau(\hat{\tau}_\nu, \hat{\eta}_a) = F_\tau(\hat{\tau}_\nu)\hat{\Theta}_{\eta_a}^{-1}\bar{\mathbf{1}} \end{array}$$

It is worth noting that $\hat{\tau}_\nu$ is the desired control input w.r.t. the new set V, computed on the estimation of the uncertainties $\hat{\eta}_a$. Hence, the control $\hat{\tau}$ is the actual torque control applied to the system.

Consider the desired torque control law (13), that take care of unknown dynamic parameter η_d with the adaptation law (15). Recalling the actual torque control $\hat{\tau} = \tau + \tilde{\tau}$ and equation (14) we obtain:

$$\begin{array}{rcl} B(\eta_d)\dot{\tilde{u}} & = & W(q)(\tau + \tilde{\tau}) - C(\eta_d)u_\tau - G_u(\eta_d) + \\ & & +\gamma^\star(\eta_d) - B(\eta_d)\dot{u}_\tau \\ & = & -K_b\tilde{u} - \Delta + Y\tilde{\eta}_d - \gamma\tilde{\eta}_d - C(\eta_d)\tilde{u} + W\tilde{\tau} \end{array} \tag{18}$$

Adding the uncertainties on the actuators η_a, the derivative of the Lyapunov function (17) is no longer defined. Hence, it is necessary to complete $\mathbf{V}_3(q, \tilde{u}, \tilde{\eta}_d)$ with:

$$\left\{ \begin{array}{l} \mathbf{V}_{\eta_a}(\eta_a, \tilde{\eta}_a) = \frac{1}{2}\bar{\mathbf{1}}^T\tilde{\Theta}_{\eta_a}^T\Theta_{\eta_a}^{-T}\Gamma_a\tilde{\Theta}_{\eta_a}\bar{\mathbf{1}} > 0 \\ \dot{\mathbf{V}}_{\eta_a}(\eta_a, \tilde{\eta}_a) = \bar{\mathbf{1}}^T\tilde{\Theta}_{\eta_a}^T\Theta_{\eta_a}^{-T}\Gamma_a\dot{\tilde{\Theta}}_{\eta_a}\bar{\mathbf{1}} \end{array} \right.$$

where $\Gamma_a > 0$ and symmetric. Analogously to (6), $\tilde{\tau} = T\tilde{\Theta}_{\eta_a}\Theta_{\eta_a}^{-1}\bar{\mathbf{1}}$, with $T = \mathrm{diag}\tau_i$. The actuators parameters adaptation law can be chosen as:

$$\dot{\tilde{\Theta}}_{\eta_a}\bar{\mathbf{1}} = \dot{\tilde{\eta}}_a = \dot{\hat{\eta}}_a = -\Gamma_a^{-1}T^TW^T\tilde{u} \tag{19}$$

and constructing the new Lyapunov function $\mathbf{V}_4(q, \tilde{u}, \tilde{\eta}_d, \eta_a, \tilde{\eta}_a) = \mathbf{V}_3(q, \tilde{u}, \tilde{\eta}_d) + \mathbf{V}_{\eta_a}(\eta_a, \tilde{\eta}_a)$, yields:

$$\begin{array}{l} \dot{\mathbf{V}}_4(q, \tilde{u}, \tilde{\eta}_d, \eta_a, \tilde{\eta}_a) = \dot{\mathbf{V}}_1(q) - \tilde{u}^TK_b\tilde{u} + \\ -\tilde{u}^TWT\tilde{\Theta}_{\eta_a}\Theta_{\eta_a}^{-1}\bar{\mathbf{1}} + \bar{\mathbf{1}}^T\tilde{\Theta}_{\eta_a}^T\Theta_{\eta_a}^{-T}\Gamma_a(-\Gamma_a^{-1}T^TW^T\tilde{u}) \\ = \dot{\mathbf{V}}_1(q) - \tilde{u}^TK_b\tilde{u} = \dot{\mathbf{V}}_2(q, \tilde{u}) \end{array}$$

that is negative semidefinite, with equilibrium point $(q, \tilde{u}, \tilde{\eta}_d, \tilde{\eta}_a) = (0, 0, \tilde{\eta}_d, \tilde{\eta}_a)$.

The most powerful feature of the proposed approach is the "separation" between the problems involved in the stabilization task, whose feasibility is achieved using backstepping techniques and computed torque frameworks. The controller design steps could be depicted briefly in what follows:

1. Design a desired control law for the kinematic nonholonomic system $u(q)$;
2. Starting from $u(q)$, design the desired torque τ for the dynamic system (see (10));
3. If there are uncertainties on the dynamic parameters η_d of the system, modify the torque control law τ with estimated values and use the dynamic parameter adaptation law (15);
4. If there are uncertainties on the actuator's parameters η_a too, add the dynamic parameter adaptation law (19).

5 Tracking control with bounded torques.

The general, modular, framework presented in the previous paragraphs is now extended to the case of constraints in the actuators torques for the case of unicycle motion. An *approaching controller* is used to minimize the gap between the controlled vehicle and the reference vehicle under a parameter dependent distance D_{min}. As the robot reaches the desired relative position, another parameter dependent distance $D_{max} > D_{min}$ is defined and a *backstepping controller* is activated that stabilizes the robot onto the desired trajectory and guarantees that the control torques are limited if the distance $e < D_{max}$, ensuring that the gap never increases besides D_{max}. It is worthwhile to note that in the region (D_{min}, D_{max}) both the controller can work and that the proposed solution involves a hystheresis on the distance e.

5.1 Backstepping Controller

Let us consider the kinematic and dynamic model of a unicycle and express the target coordinates in the mobile vehicle reference system, then the error dynamics $\mathbf{e} = [e_1, e_2, e_3]^T$ are given by the equations:

$$\begin{cases} \dot{e}_1 = v_r \cos(e_3) - v + e_2\omega \\ \dot{e}_2 = v_r \sin(e_3) - e_1\omega \\ \dot{e}_3 = \omega_r - \omega \end{cases} \Rightarrow \dot{\mathbf{e}} = f(\mathbf{e}) + G(\mathbf{e})u \tag{20}$$

where v and ω are the forward and steering velocities. Dynamics are added by considering the equations:

$$\begin{cases} \dot{v}(\mathbf{e}) = \frac{\tau_v}{m} \\ \\ \dot{\omega}(\mathbf{e}) = \frac{\tau_\omega}{I_z} \end{cases}$$

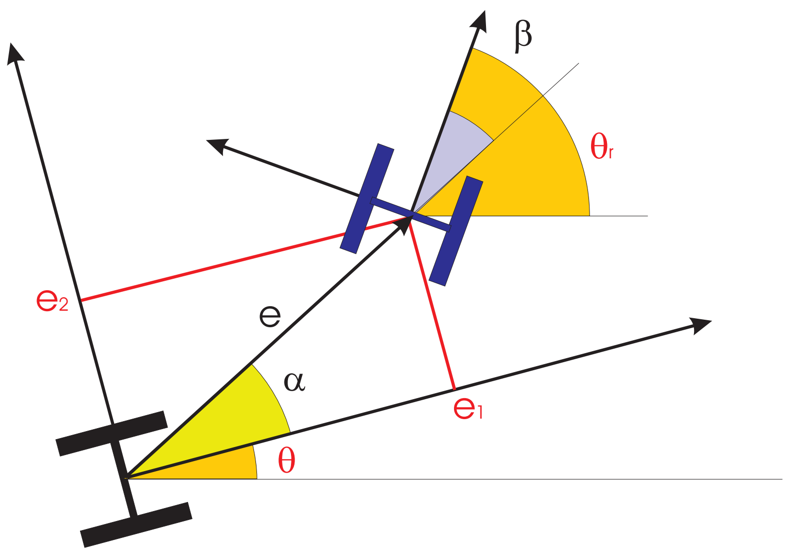

Figure 1: Trajectory tracking problem geometry.

where m and I_z are the mass and the momentum of inertia respectively.

Using the control Lyapunov function

$$V_1 = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \left(1 - \cos\left(e_3\right)\right) \tag{21}$$

we are able to synthesize two kinematic control laws for the forward and steering velocities, $\bar{v}$ and $\bar{\omega}$ respectively

$$\begin{array}{rcl} \bar{v}(\mathbf{e}) & = & \frac{2}{\pi} V_{\max} \arctan\left(e_1\right) - v_r \cos\left(e_3\right) \\ \bar{\omega}(\mathbf{e}) & = & \omega_r + \frac{1}{K_{e_3}} e_2 v_r + \frac{1}{K_{s_3}} \sin\left(e_3\right) \end{array} \tag{22}$$

that make $\dot{V}_1$ negative definite. It is easy to notice that the surge velocity $\bar{v}$ is surely bounded by $V_{max}+v_r$, while $\bar{\omega}$ maximum value depends on the positive constants K_{e_3}, K_{s_3}, but also on the distance, through the error variable e_2.

Adding dynamics, a new control Lyapunov function is obtained as:

$$V_1^d\left(e, v, \omega\right) = V_1 + \frac{1}{2}\left(v - \bar{v}\right)^2 + \frac{1}{2}\left(\omega - \bar{\omega}\right)^2$$

that yields

$$\begin{array}{rcl} \tau_v & = & m(-K_{bv}\left(v - \bar{v}\right) + \frac{\partial \bar{v}}{\partial e}\left(f\left(e\right) + G\left(e\right)u\right) + \\ & & +\frac{\partial V_1}{\partial e} g_v\left(e\right)) \\ \tau_\omega & = & I_z(-K_{b\omega}\left(\omega - \bar{\omega}\right) + \frac{\partial \bar{\omega}}{\partial e}\left(f\left(e\right) + G\left(e\right)u\right) + \\ & & +\frac{\partial V_1}{\partial e} g_\omega\left(e\right)) \end{array} \tag{23}$$

where K_{bv} and $K_{b\omega}$ are positive constants, $\bar{v}$ and $\bar{\omega}$ are the reference velocities provided by the kinematic controller, and $f(\mathbf{e})$ and $G(\mathbf{e})$ are again the vector fields of the kinematic model (20).

We are now interested in finding a maximum for the two torques (23):

$$\begin{array}{l} |\tau_{v_{\max}}| = m(2K_{bv}\,|v_{\max}| + |v_{r_{\max}}|\,(|\omega_{\max}| + \\ + |\omega_{r_{\max}}|) + |v_{r_{\max}}| + |v_{\max}| + |d||\omega_{\max}| + |d|) = \\ = T_{v_1} + d \cdot T_{v_2} \\ \\ |\tau_{\omega_{\max}}| = I_z(2K_{b\omega}\,|\omega_{\max}| + K_{e_3}^{-1}(v_{r_{\max}}^2 + \\ + |v_{r_{\max}}|\,|\omega_{\max}| \cdot d) + K_{s_3}^{-1}\,(|\omega_{\max}| + |\omega_{r_{\max}}|) + \\ + K_{e_3}) = T_{\omega_1} + d \cdot T_{\omega_2} \end{array} \tag{24}$$

As shown in equations (24), the maximum value of the torques are given by a linear relation $|\tau_{\max}| = T_1 + d \cdot T_2$, where T_1 and T_2 are functions of the vehicle maximum velocity and inertial parameters. The lower limit of the torque value is T_1, hence the problem to solve is to find the maximum value of d in order to constraint the torque into the range $[0, T_1 + \Delta_{\max}]$, with $\Delta_{\max} > 0$. Since the torque controls critically depend on the distance between the vehicle and the desired reference vehicle, before the resulting backstepping torque controls can be applied to the system, an additional controller that approaches the desired reference vehicle is adopted.

5.2 Approaching Controller

This controller is meant to drive the vehicle inside the range where the above controller works with constrained torques. Consider a new state space q for the vehicle (see again figure 1), where $q = [e, \alpha, \beta]^T$ with e the distance vector between the vehicle and the target and with α and β angles between vector e and the relative direction of each vehicle. The kinematic model of the variables q is then:

$$\begin{cases} \dot{e} = -v\cos\alpha + v_r\cos\beta \\ \dot{\alpha} = -\omega + v\frac{\sin\alpha}{e} + v_r\frac{\sin\beta}{e} \\ \dot{\beta} = \omega_r - v\frac{\sin\alpha}{e} - v_r\frac{\sin\beta}{e} \end{cases}$$

Considering the control Lyapunov Function

$$V_2(q) = \frac{1}{2}\alpha^2 + \frac{1}{2}\ln(1 + e^2),$$

its time derivative:

$$\begin{array}{rcl} \dot{V}_2(q) & = & \alpha\dot{\alpha} + \frac{e}{1+e^2}\dot{e} = \alpha\left(-\omega + v\frac{\sin\alpha}{e} + v_r\frac{\sin\beta}{e}\right) + \\ & & + \frac{e}{1+e^2}(-v\cos\alpha + v_r\cos\beta) \end{array}$$

and by substituting the controls $\bar{v}$ and $\bar{\omega}$:

$$\begin{cases} \bar{v} = \mathrm{sat}(e)\cos\alpha \\ \omega = K_\alpha \alpha + v\frac{\sin\alpha}{e} + v_r\frac{\sin\beta}{e} \end{cases}$$

where $\mathrm{sat}(e)$ is a generic saturation function. The function $\dot{V}_2$ is n.d. if $K_\alpha > \frac{v_{r\max}}{2}$.

As in (23), backstepping techniques lead to the control laws for the vehicle's dynamic model, that can be maximized as in (24):

$$\begin{aligned} |\tau_{v_{\max}}| &= m(2K_{bv}|v_{\max}| + \tfrac{R_{\max}}{\pi}(|v_{r_{\max}}| + |v_{\max}|) + \\ &\quad +\pi(1 + K_\alpha + \tfrac{1}{2\pi})) \\ |\tau_{\omega_{\max}}| &= I_z(2K_{b\omega}|\omega_{\max}| + (|v_{r_{\max}}| + |v_{\max}|)^2 + \\ &\quad +K_\alpha(|v_{\max}| + K_\alpha) + |v_{r_{\max}}|(|\omega_{r_{\max}}| + \\ &\quad +|v_{\max}| + |v_{\max}|) + \pi) \end{aligned} \tag{25}$$

The control laws (25) allow to bound the control torques while the vehicle approaches the region where the controller (23) is able to track the targret and respect the torques constraints.

5.3 Switching Control Law

We are now interested in fixing the value of the switching distance D_{min} from the approaching to the backstepping controller. The occurring of the switch at a distance D_{min} ensures that the distance will not exceed D_{max} and therefore the torque constraints will be respected. Let us consider an isosurface of the Lyapunov function V_1. The projections on the (e_1, e_2) plane for different e_3 angles are concentric circles (see figure 2). From Lyapunov theory, a system trajectory originated inside an isosurface with n.d. time derivative is bounded in the same isosurface. This means that, in the worst case, when the backstepping controller is activated, a trajectory starting at a distance D_{min} from the origin of the (e_1, e_2) plane will never exceed the D_{max} distance, that is the radius of the isosurface computed for e_3 equals to zero. An idea of the convergence of the proposed control method is given considering that the backstepping controller is globally asymptotically stable and that multiple switches are avoided as soon as the distance between the target and the vehicle is less than D_{min}. The approaching controller ensures that the switch will be fired by decreasing the distance between the target and the vehicle.

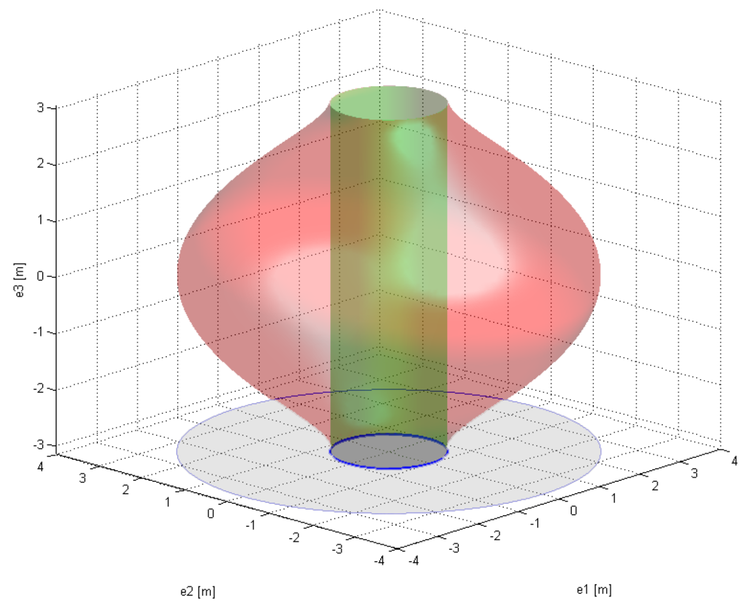

Figure 2: A 3D representation of the isosurface.

6 Simulation results

6.1 Regulation with kinematic uncertainties and limited velocity

As an application example of the method explained previously, the unicycle vehicle, with the kinematic control law reported in (Murrieri *et al.*, 2003), is adopted. In what follows, the control law is generically referred as $u(q) = [u_1(q), u_2(q)]^T \in U$, where $q = [\rho, \phi, \beta]^T$. The control laws are:

$$\begin{cases} v &= \rho \cos\beta \\ \omega &= \frac{\phi \sin\beta}{\lambda\rho\beta} v + \frac{\sin\beta}{\lambda\rho} v + \sigma\beta \end{cases} \tag{26}$$

where $\sigma > 0$ and $\lambda > 0$.

Let $u = [v, \omega]^T = [u_1, u_2]^T$ be the available controls, i.e. the forward and steering velocities of the vehicle respectively. If the uncertain parameters related to the actuators are the wheel radius R and the inter axle length L, it is possible to express the new set of input controls $u_\nu = [\omega_r, \omega_l]^T = [u_{\nu_1}, u_{\nu_2}]^T \in V$ as the angular velocity of the vehicle's wheels, on the right and left side of the robot respectively. Indeed:

$$\left\{\begin{array}{l} v = \frac{\omega_r+\omega_l}{2}R \\ \omega = \frac{\omega_r-\omega_l}{L}R \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \omega_r = \frac{v}{R} + \frac{L\omega}{2R} \\ \omega_l = \frac{v}{R} - \frac{L\omega}{2R} \end{array}\right. .$$

Choosing the uncertain parameter as $\eta_a = [1/R, L/R]^T = [\eta_{a_1}, \eta_{a_2}]^T$, follows that:

$$u_\nu = \tilde{F}^{-1}(u, \eta_a) = \left[\begin{array}{cc} u_1 & \frac{u_2}{2} \\ u_1 & -\frac{u_2}{2} \end{array}\right] \left[\begin{array}{cc} \eta_{a_1} & 0 \\ 0 & \eta_{a_2} \end{array}\right] \bar{\mathbf{1}}$$

The limited velocity constraint is defined directly in the perfectly known control law. The admissible velocities are limited by $|v| \leq v_{max}$ and $|\omega| \leq \omega_{max}$. Noting that the dynamic of the distance is nonincreasing ($\dot{\rho} \leq 0$) along the controlled trajectories of the vehicle, the linear velocity v is bounded once

$$v = v_{max}\frac{\rho}{\rho_0}\cos\beta \tag{27}$$

where ρ_0 is the distance in the initial position (at $t = t_0$). Substituting (27) in (26), the angular velocity constraint is clearly verified if

$$\frac{v_{max}}{\rho_0}\frac{1}{\lambda}\left|\sin\beta\cos\beta\frac{\phi+\lambda\beta}{\beta}\right| + \sigma\,|\beta| < \omega_{max}$$

since $\lambda > 0$ and $\sigma > 0$. Noting that $\phi \in (-\pi, \pi]$ and $\beta \in (-\pi, \pi]$, it is straightforward that

$$k = \max\left(\left|\sin\beta\cos\beta\frac{\phi+\lambda\beta}{\beta}\right|\right) > 0$$

is known and easily numerically computable. Hence, the constrint on ω is satisfied if:

$$\frac{v_{max}}{\lambda\rho_0}k + \sigma\pi < \omega_{max}$$

that further impose a lower limit on ρ_0

$$\rho_0 > \frac{k v_{max}}{\lambda(\omega_{max} - \sigma\pi)}$$

Since $\lambda > 0$ and $\rho_0 > 0$, $\sigma < \omega_{max}/\pi$. To fulfill both the constraints on the limited velocities, it is trivial that

$$\rho_0 = \max\left\{\rho(t_0), \frac{k v_{max}}{\lambda(\omega_{max} - \sigma\pi)}\right\}$$

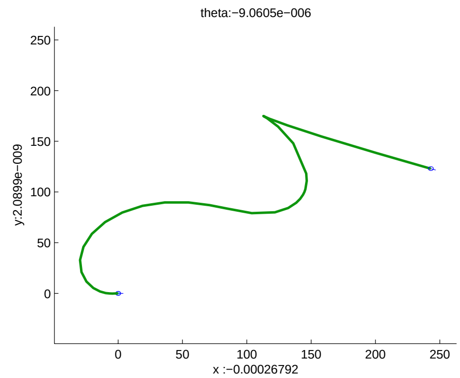

Figure 3: Vehicle manoeuvre during a docking operation with unknown parameters and limited velocities.

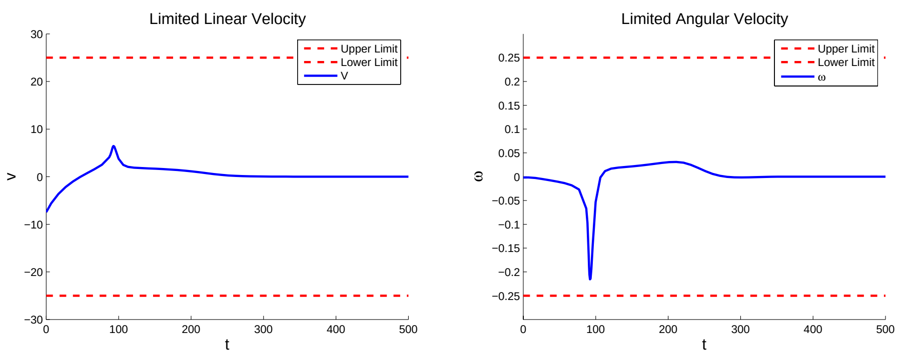

Figure 4: Computed controls: linear velocity v (left), angular velocity ω (right). Each figure reports the velocity limits too.

Once the aforementioned velocity limits are extended to the uncertain model, the limit should be divided by two.

In figure 3 is depicted a parking trajectory when the robot is placed in $q = [270, 3.6, 3.95]^T$ and the vehicle parameters are set to $R = 100$ and $L = 500$ (the same data as the previous example). The estimated parameters value is $\hat{R} = 190$ and $\hat{L} = 150$. The velocity limits are $(v_{max}, \omega_{max}) = (50\ mm/sec,\ 1/2\ rad/sec)$, while the scaling factor $\rho_0 = 660$. The controller parameter $\lambda = 0.04$. The parking problem is solved using the adaptive controller with limited velocity.

In figure 4, the computed controls are reported (the linear velocity v, left, and the angular velocity ω, right). Each figure depicts also the velocity limits.

6.2 Regulation with dynamic uncertainties

Consider the dynamic system of the unicycle, extension of kinematic system:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\phi} \\ \dot{\beta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\rho\cos\beta v \\ \sin\beta v \\ \sin\beta v - \omega \\ v^2\cos\beta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\rho m} \\ 0 \end{bmatrix} \tau_v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{I} \end{bmatrix} \tau_\omega \tag{28}$$

where $\tau = [\tau_v, \tau_\omega]^T$ are respectively the linear and angular control torques. m is the mass of the vehicle, whose center of mass is on the mid point of the wheel axle. I is the inertia momentum with respect to the vertical axis, constantly perpendicular to the plane of motion. Supposing that the robot navigates on a plane, $C(\dot{q}, q)$ and $G(q)$ matrices are zero.

Using the kinematic control law (26), the positive definite Lyapunov function:

$$\mathbf{V_1}(q) = \frac{1}{2}q^T\Lambda q > 0, \tag{29}$$

where $\Lambda > 0$ is a symmetric matrix, gives a negative definite time derivative.

Let $\eta_d = [1/m, 1/I]^T$, $u = [v, \omega]^T$ and:

$$\begin{aligned} \gamma(q) &= \begin{bmatrix} u_1^2\cos\beta & 0 \\ 0 & 0 \end{bmatrix}, B(q,\eta_d) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}, \\ W(q,\eta_d) &= \begin{bmatrix} \frac{1}{\rho} & 0 \\ 0 & 1 \end{bmatrix}, K_b = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \end{aligned} \tag{30}$$

In figure 5 is depicted a parking trajectory when the robot is placed in $q = [35.5,\ 1.80,\ -1.50]^T$ and the dynamic parameters are set to $m = 10$ and $I = 1$. The estimated parameters value is $\hat{m} = 21$ and $\hat{I} = 7$. The controller parameter $\lambda = 1/2$. On the left, the parking problem is solved using the adaptive controller while, on the right side of the figure, the parking problem is carried out with the backstepping controller $u(q)$, without any parameter adaptation. The figure 6 depicts the computed torque controls for the linear (left) and angular (right) velocity for adaptive and dynamic controller obtained using only backstepping techniques.

6.3 Regulation with dynamic and kinematic uncertainty

Let $\tau = [\tau_v, \tau_\omega]^T = [\tau_1, \tau_2]^T$ be the available torque controls, i.e. the linear and angular torques of the vehicle respectively, and $\tau_\nu = [\tau_r, \tau_l]^T = [\tau_{\nu_1}, \tau_{\nu_2}]^T$

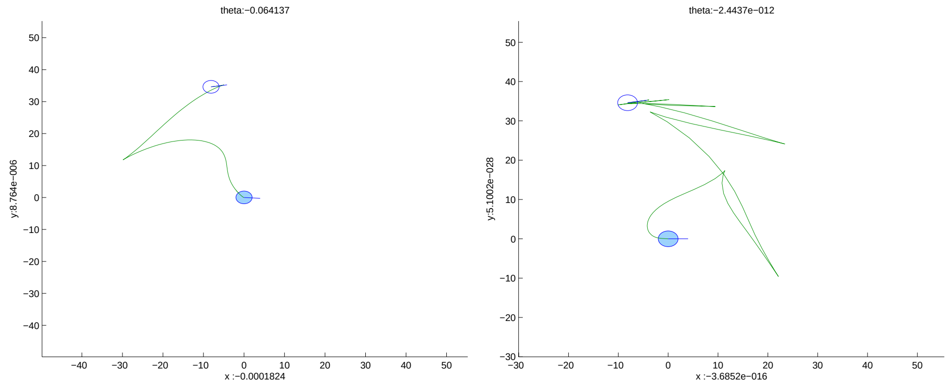

Figure 5: Vehicle manoeuvre during a docking operation with unknown dynamic parameters. Both the adaptive (left) and backstepping (right) controller behavior is reported.

be the torques of the vehicle's wheels, on the right and left side of the robot respectively:

$$\begin{cases} \tau_v = \frac{\tau_r+\tau_l}{2} R \\ \tau_\omega = \frac{\tau_r-\tau_l}{L} R \end{cases} \Leftrightarrow \begin{cases} \tau_r = \frac{\tau_v}{R} + \frac{L\tau_\omega}{2R} \\ \tau_l = \frac{\tau_v}{R} - \frac{L\tau_\omega}{2R} \end{cases} .$$

In figure 7 is depicted a parking trajectory when the robot is placed in $q = [380, 0.92, 3.97]^T$, with the mass $m = 10$ and the inertia momentum $I = 1$ and with the actuator's parameters set to $R = 100$ and $L = 500$. The dynamic estimated parameters $\hat{\eta}_d = [\hat{m}, \hat{I}]^T = [76, 4.5]^T$ and the actuator's estimated parameters $\hat{\eta}_a = [\hat{R}, \hat{R}/\hat{L}]^T = [174, 0.26]^T$ ($\hat{L} = 664$). The controller parameter $\lambda = 1/2$. The simulation has been carried out for $20sec$. On the left, the parking problem is solved using the adaptive controller while, on the right side of the figure, the parking problem is carried out with the native controller $\Psi(q)$, without any parameter adaptation. In figure 8, the control error with respect to the desired controls (τ_v, τ_ω) computed with perfect actuator's parameter knowledge $\tilde{\eta}_a = 0$, is reported (up, left and right figures respectively) for adaptive and native controller. The figure 8 depicts also the computed controls for the right and left side of the vehicle (down, left and right figures respectively), for adaptive and native controller.

6.4 Tracking control with bounded torques

Simulation results for the tracking control of a target vehicle with bounded torques are presented. In what follows, the reference vehicle describes a circle while the controlled vehicle starts from differentpositions.

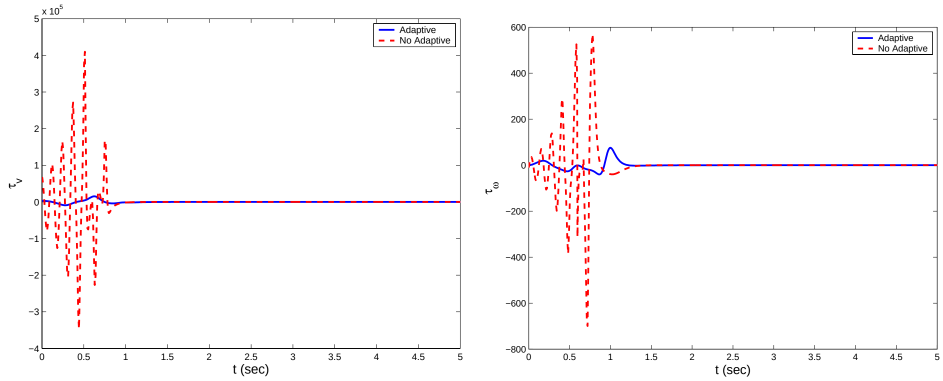

Figure 6: Computed torque controls. Both the linear (left) and angular (right) controls are reported.

The vehicle's parameters are $m = 5kg$, $I_z = 1kgm^2$, $v_{\max} = 2m/s$, $\omega_{\max} = 1.5rad/s$, while the maximum reference velocities are set to $v_{r_{max}} = 1m/s$ and $\omega_{r_{max}} = 0.2rad/s$. The torque bounds are $|\tau_{v_{max}}| = 100Nm$ and $|\tau_{\omega_{max}}| = 50Nm$. In order to respect these constraints, control parameters are selected as follows: $D_{min} = 1$, $K_{e_3} = 3$, $K_{s_3} = 10$ and $D_{max} = 3.6$. The backstapping gains are $K_{bv} = 1/6$, $K_{b\omega} = 5/3$, $K_\alpha = 1/2$ and the saturation function $|\text{sat}(\pm\infty)| = 3$ in order to respect (24).

Substituting the previous parameters in (24) and (25) the constraints are satisfied since $|\tau_{v_{max}}| = 60Nm$ and $|\tau_{\omega_{max}}| = 30Nm$.

In figure 9 the trajectories of the controlled robot starting from $q = [D\cos\delta, D\sin\delta, \delta]^T$, with $D = 10$ and $\delta \in (0, \pi/2, \pi, -\pi/2)$ are depicted. The target starts from $q_r = t[0, -5, 0]^T$. The corresponding computed torque controls are reported in figure 10.

Starting from the same intial condition for the controlled and reference robots, computed torque controls for the backstepping control law acting alone are reported in figure 11. It is worthwhile to note that in this second case the control torques are one magnitude greater than the other computed with the switching control law.

7 Conclusions

Nonlinear adaptive control laws for generic kinematic nonholonomic systems in the presence of actuators limits and uncertainties have been derive. An extension to uncertain dynamic systems using backstepping techniques and control Lyapunov functions has been used. It has been shown that it is

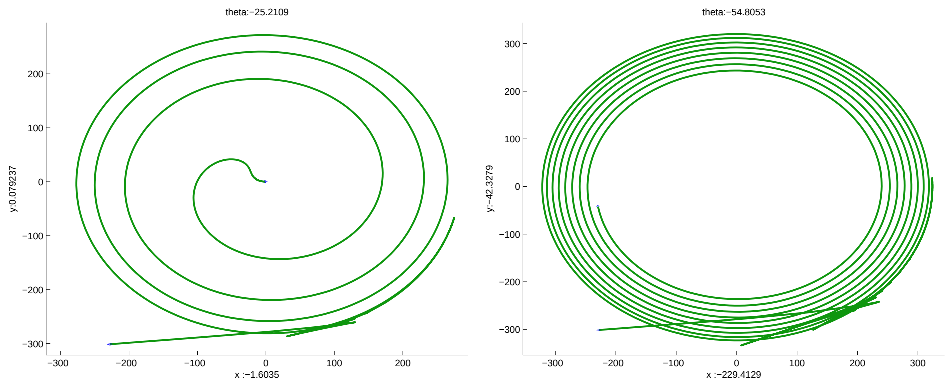

Figure 7: Vehicle manoeuvre during a docking operation with both unknown dynamic and actuator's parameters. Both the adaptive (left) and native (right) controller behavior is reported.

possible to obtain a control Lyapunov function in a modular way, starting from a stabilizing law for the kinematic, perfectly known model. Our effort has been devoted to bound the control inputs of the kinematic and dynamic system, in order to avoid actuators saturation.

Simulation results has been shown, showing a vehicle manoeuvres during a parking operation with unknown parameters and limited velocities and vehicle manoeuvres during a tracking operation with limited torques. Our future efforts will focus on merging the previous results and obtaining smooth control laws for the set-point and tracking control problems.

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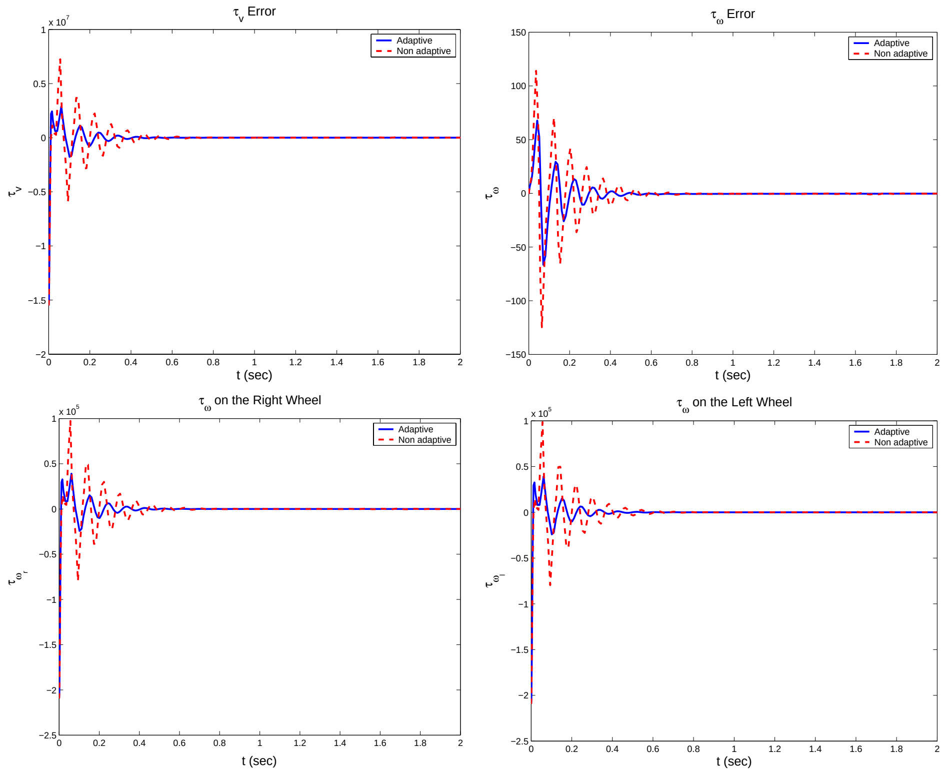

Figure 8: Control error with respect to the desired one (up) and computed controls for the right and left side of the vehicle (down). Both the adaptive and native controller torques are reported in each graph. Only significative data are reported ($2sec$ of the whole $20sec$ simulation time).

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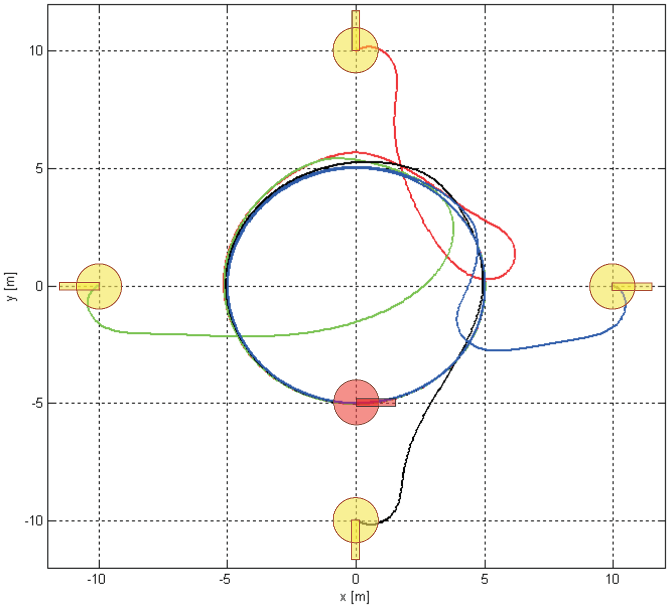

Figure 9: Trajectories of mobile robot at different starting position.

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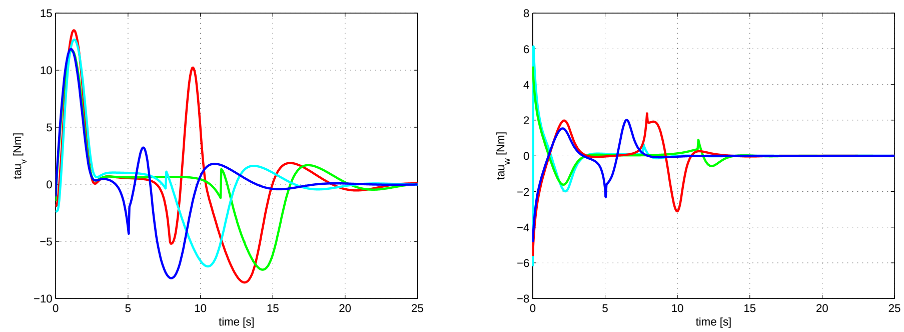

Figure 10: Computed torque controls: linear (left) and angular (right) controls.

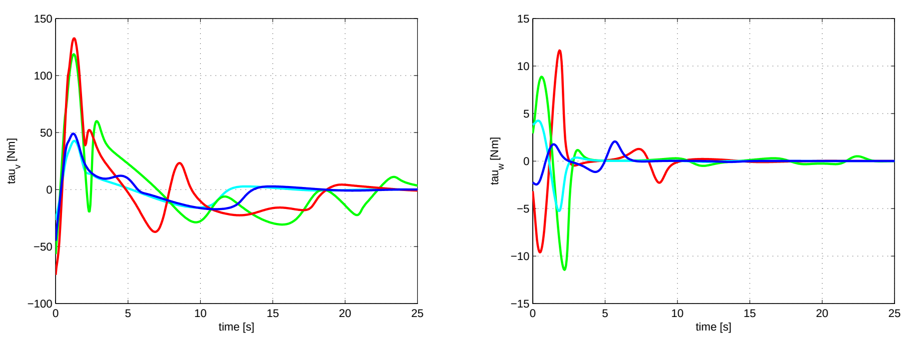

Figure 11: Computed torque controls: linear (left) and angular (right) controls for the backstapping control acting alone.