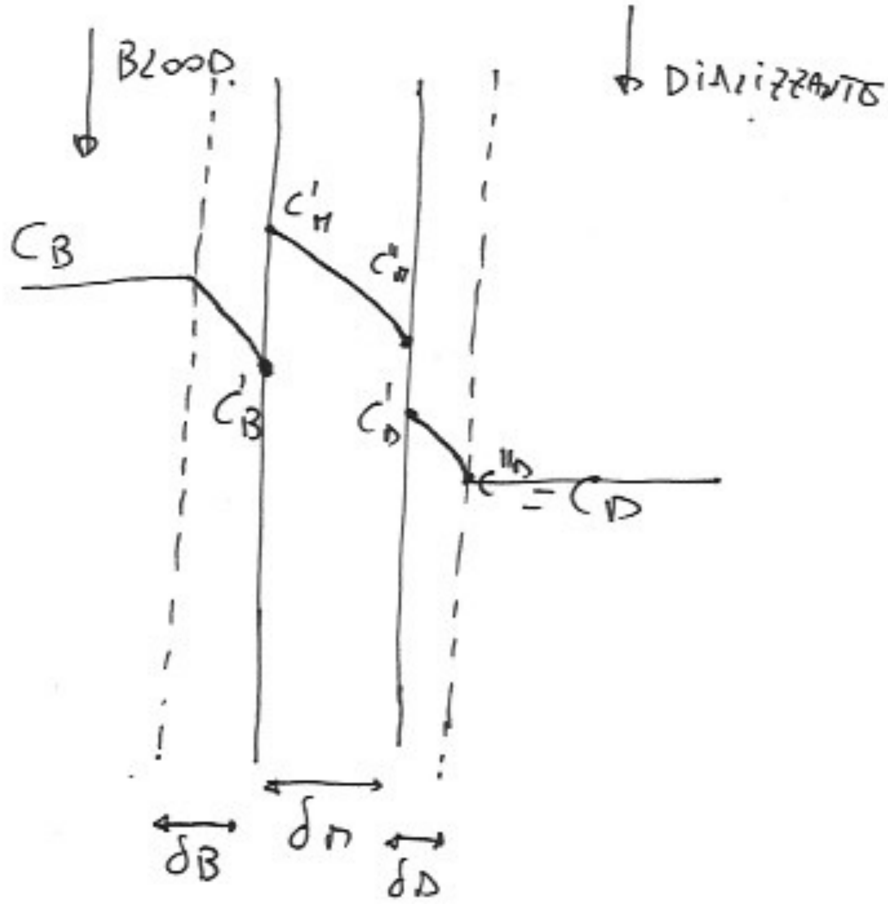



$$N = D_B \frac{(C_B - C'_B)}{\delta_B} = D_M \frac{(C'_M - C''_M)}{\delta_M} = D_D \frac{(C'_D - C_D)}{\delta_D}$$

$$C_B - C_D = (C_B - C'_B) + C'_B + C'_D - C'_D - C_D =$$

$$= (C_B - C'_B) + (C'_D - C_D) + (C'_B - C'_D)$$

$$C_B - C'_B = \frac{N\delta_B}{D_B} \quad (C'_D - C_D) = \frac{N\delta_M}{D_M}$$

$$\alpha = \frac{C'_M}{C'_B} = \frac{C''_M}{C'_D}$$

$$(C'_B - C'_D) = \frac{C'_M}{\alpha} - \frac{C''_M}{\alpha} = \frac{1}{\alpha}(C'_M - C''_M) = \frac{1}{\alpha}\frac{N\delta_M}{D_M}$$

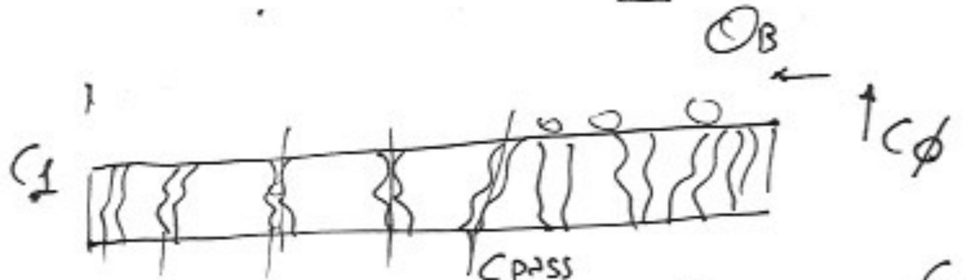
$$C_B - C_D = \frac{N\delta_B}{D_B} + \frac{N\delta_D}{D_D} + \frac{N\delta_m}{\alpha D_m}$$

$$N = \frac{(c_B - c_D)}{\frac{\delta_B}{D_B} + \frac{\delta_D}{D_D} + \frac{\delta_m}{\alpha D_m}} = K\,(c_B - c_D)$$

$$K = \frac{1}{\frac{\delta_B}{D_B} + \frac{\delta_D}{D_D} + \frac{\delta_m}{\alpha D_m}} = \text{coeff. di trasp. di membrana}$$

$$\frac{1}{K} = R = \frac{\delta_B}{D_B} + \frac{\delta_D}{D_D} + \frac{\delta_m}{\alpha D_m} \Rightarrow$$

$$R_m = \frac{\delta_m}{\alpha D_m} \qquad \boxed{\alpha \to 1} \qquad \underline{D_m}$$

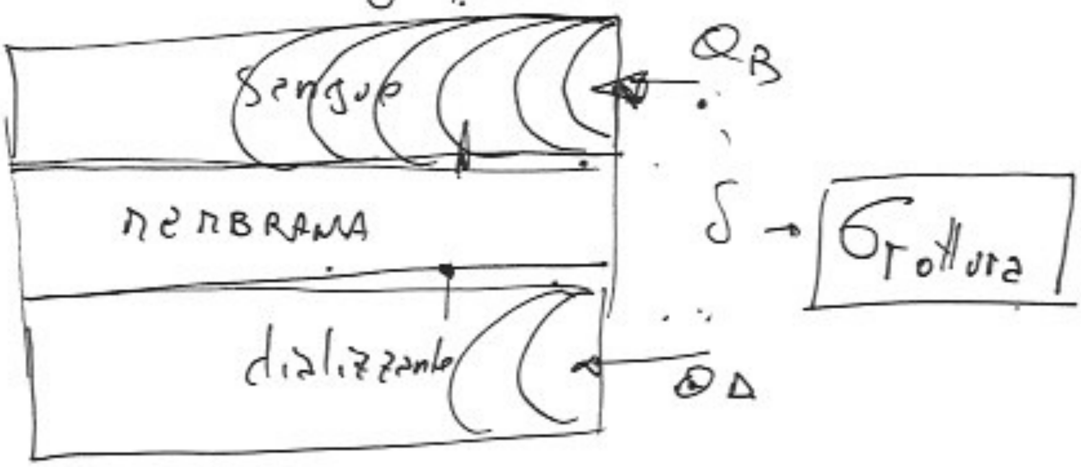



$$\Delta c = c_0 - c_1 = c_{ass} + c_{pass} \qquad c_{ass} = \underline{\Delta c - c_{pass}}$$

$$\frac{c_{ass}}{c_\phi} = \alpha \qquad J = D_m \frac{\delta c}{dx} \qquad Q_B = D_m \frac{c_0 - c_{pass}}{\delta_m}$$

$$D_m^1 \frac{C_0 - C_{pass}}{\delta m} = J$$

$$D_m^2 \frac{C_1 - C_{pass}}{\delta m} = J$$

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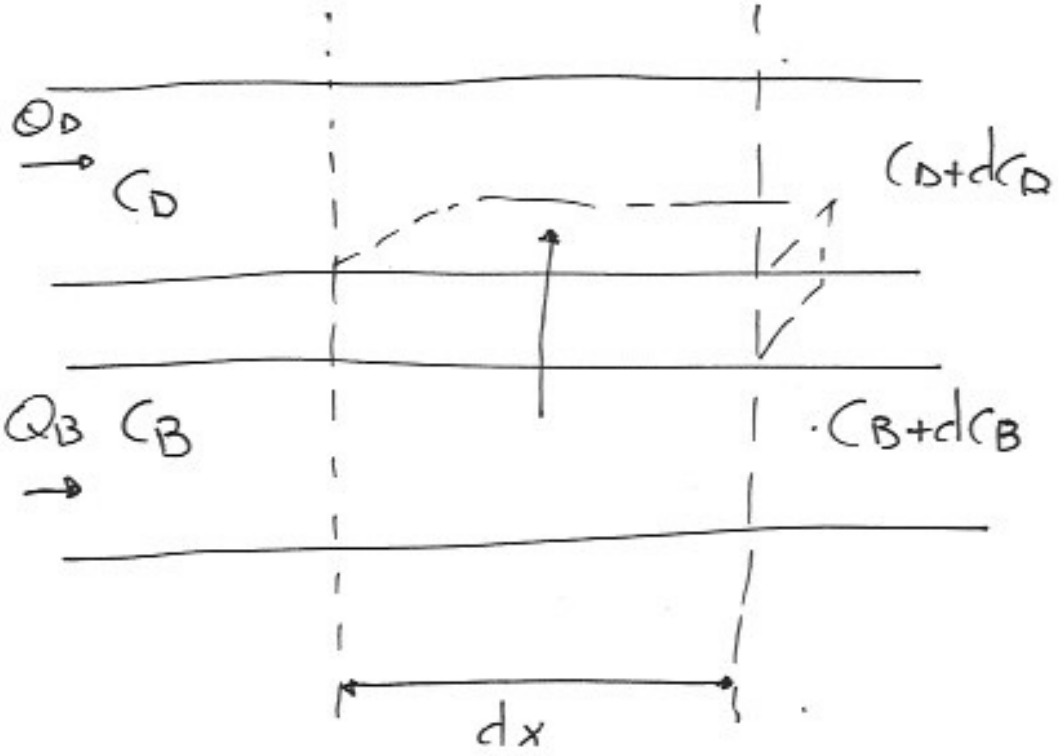
$R_m = \frac{\delta m}{2 D_m}$ $\quad$ $Q_B = 200 \frac{ml}{min}$


co-corrente


contro corrente


$Q_D >> Q_B$

flusso misto




$$dW = \underbrace{K(c_B - c_D)\,dA}_{1} = \underbrace{-Q_B\,dc_B}_{2} = \underbrace{Q_D\,dc_D}_{3}$$

$$\begin{cases} \dfrac{dc_B}{c_B - c_D} = -\dfrac{K\,dA}{Q_B} & (4) \\[2ex] \dfrac{dc_D}{c_B - c_D} = \dfrac{K\,dA}{Q_D} & (5) \end{cases} \qquad dN = \frac{dW}{dA}$$

$$\frac{dc_B}{c_B - c_D} - \frac{dc_D}{c_B - c_D} = -K\,dA\left[\frac{1}{Q_B} + \frac{1}{Q_D}\right]$$

$$\frac{d(c_B - c_D)}{c_B - c_D} = -K\,dA\left[\frac{1}{Q_B} + \frac{1}{Q_D}\right]$$

$$\int_i^{o} \frac{d(c_B - c_D)}{c_B - c_D} = + \int_A K\, dA \left[\frac{1}{Q_B} + \frac{1}{Q_D}\right]$$ (5)

$$\ln \frac{(c_{Bi} - c_{Di})}{(c_{Bo} - c_{Do})} = + K \left[\frac{1}{Q_B} + \frac{1}{Q_D}\right] A$$

$$dW = K(c_B - c_D)\, dA = -Q_B\, dc_B = Q_D\, dc_D$$

(1) (2) (3)

$$dW = -Q_B\, dc_B \rightarrow \int dW = -Q_B \int_i^o dc_B$$

$$dW = Q_D\, dc_D \qquad \int dW = Q_D \int_i^o dc_D.$$

$$W = -Q_B (c_{Bo} - c_{Bi}) \qquad Q_B = -\frac{W}{(c_{Bo} - c_{Bi})}$$

$$W = Q_D (c_{Do} - c_{Di}) \qquad Q_D = \frac{W}{(c_{Do} - c_{Di})}$$

$$\ln \frac{(c_{Bi} - c_{Di})}{(c_{Bo} - c_{Do})} = KA \left[-\frac{[c_{Bo} - c_{Bi}]}{W} + \frac{(c_{Do} - c_{Di})}{W}\right]$$

$$\ln \frac{(c_{Bi} - c_{Di})}{(c_{Bo} - c_{Do})} = \frac{KA}{W} \cdot \left[(c_{Bi} - c_{Di}) - (c_{Bo} - c_{Do})\right]$$

$$W = KA \frac{[(c_{Bi} - c_{Di}) - (c_{Bo} - c_{Do})]}{\ln \frac{(c_{Bi} - c_{Di})}{(c_{Bo} - c_{Do})}}$$

$$A_{OTT} = A_{mean} = mean(A_i)$$

$$\begin{cases} Urea \\ Acido\,urico \\ Na^+ \\ Glucosio \end{cases} \quad A_{mean} = \frac{A_{urea} + A_{acido\,urico} + A_{Na^+} + A_{glucosio}}{4}$$

$$K = \frac{1}{R} \qquad R = \frac{\delta\pi}{\alpha D\pi} \qquad A$$

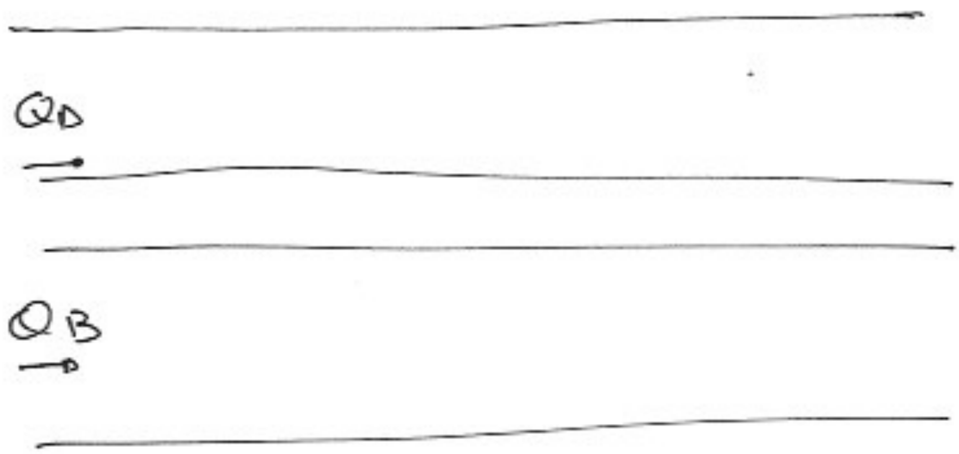
$$D^* = Dialysance = Q_B \frac{(C_{Bi} - C_{Bo})}{(C_{Bi} - C_{Di})} = \frac{W}{(C_{Bi} - C_{Di})}$$

$$C^* = Clearance = Q_B \left.\frac{(C_{Bi} - C_{Bo})}{(C_{Bi} - C_{Di})}\right|_{C_{Di}=\phi} = Q_B \frac{(C_{Bi} - C_{Bo})}{C_{Bi}} =$$

$$= \frac{W}{C_{Bi}}$$

$$E = Extractio\ Ratio = \left.\frac{D}{Q_B}\right|_{C_{Di}=0} = \frac{C_{Bi} - C_{Bo}}{C_{Bi}} = 1 - \frac{C_{Bo}}{C_{Bi}}$$

Co-Corrente




$$dW = K(C_B - C_D)\,dA = -Q_B\,dC_B = Q_D\,dC_D$$

$$W = -Q_B\,(C_{Bo} - C_{Bi}) \qquad W = Q_D\,(C_{Do} - C_{Di})$$

$$\frac{W}{Q_B} = -C_{Bo} + C_{Bi} \qquad \frac{W}{Q_D} = C_{Do} - C_{Di}$$

$$C_{Bo} = C_{Bi} - \frac{W}{Q_B} \qquad C_{Do} = C_{Di} + \frac{W}{Q_D}$$

$$W = \frac{K\,A\left[(C_{Bi} - C_{Di}) - (C_{Bo} - C_{Do})\right]}{\ln\dfrac{(C_{Bi} - C_{Di})}{(C_{Bo} - C_{Do})}}$$

$$W = \frac{K\,A\left[\cancel{C_{Bi}} - \cancel{C_{Di}} - \cancel{C_{Bi}} + \frac{W}{Q_B} + \cancel{C_{Di}} + \frac{W}{Q_D}\right]}{\ln\dfrac{(C_{Bi} - C_{Di})}{\left(C_{Bi} - \frac{W}{Q_B} - C_{Di} - \frac{W}{Q_D}\right)}}$$

$$\cancel{W} = \frac{KA\cancel{W}\left[\frac{1}{Q_B}+\frac{1}{Q_D}\right]}{\ln \frac{(C_{Bi}-C_{Di})}{(C_{Bi}-C_{Di})-W\left[\frac{1}{Q_B}+\frac{1}{Q_D}\right]}}$$

$$\ln \frac{(C_{Bi}-C_{Di})}{(C_{Bi}-C_{Di})-W\left[\frac{1}{Q_B}+\frac{1}{Q_D}\right]} = KA\left[\frac{1}{Q_B}+\frac{1}{Q_D}\right]$$

$$\ln \frac{1}{1-\frac{W}{(C_{Bi}-C_{Di})}\frac{1}{Q_B}\left(1+\frac{Q_B}{Q_D}\right)} = \frac{KA}{Q_B}\left[1+\frac{Q_B}{Q_D}\right]$$

$$Z = \frac{Q_B}{Q_D} \qquad N_T = \frac{KA}{Q_B}$$

$$\ln \frac{1}{1-E(1+z)} = N_T(1+z)$$

$$\ln 1-E(1+z) = -N_T(1+z)$$

$$1-E(1+z) = e^{-N_T(1+z)} \Rightarrow E(1+z) = -e^{-N_T(1+z)}+1$$

$$E_{c.c.} = \frac{1-e^{-N_T(1+z)}}{1+z}$$

$$E_{c.c.} = \frac{1 - e^{-N_T(1+z)}}{(1+z)} \simeq 1 - e^{-N_T}$$

$$z = \frac{Q_B}{Q_D} \simeq \emptyset \qquad N_T = \frac{KA}{Q_B}$$

$$Q_D >> Q_B$$

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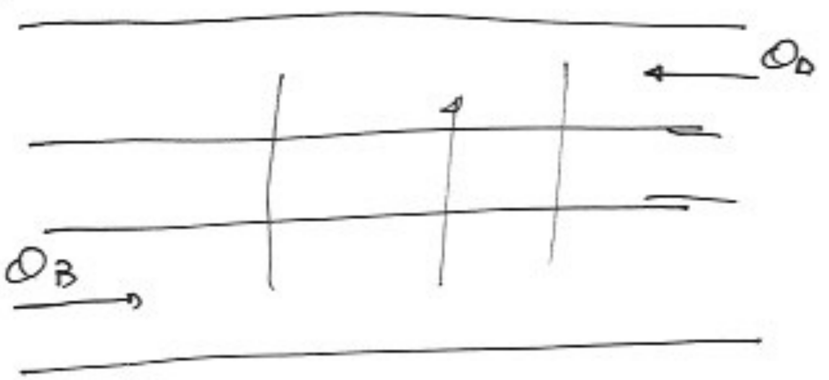
Contro-Corrente




$$W_{c.c.} = KA \frac{(C_{Bi} - C_{Di}) - (C_{Bo} - C_{Do})}{\ln \frac{(C_{Bi} - C_{Di})}{(C_{Bo} - C_{Do})}}$$

$$W_{coc} = KA \frac{(C_{Bi} - C_{Do}) - (C_{Bo} - C_{Di})}{\ln \frac{(C_{Bi} - C_{Do})}{(C_{Bo} - C_{Di})}}$$

$$W = -Q_B (C_{Bo} - C_{Bi}) \rightarrow -\frac{W}{Q_B} = C_{Bo} - C_{Bi} \quad C_{Bo} = C_{Bi} - \frac{W}{Q_B}$$

$$W = Q_D (C_{Do} - C_{Di}) \rightarrow \frac{W}{Q_D} = C_{Do} - C_{Di} \quad C_{Do} = C_{Di} + \frac{W}{Q_D}$$

$$W_{coc} = \frac{KA\left[\cancel{C_{Bi}} - \cancel{C_{Di}} - \frac{W}{Q_D} - \cancel{C_{Bi}} + \frac{W}{Q_B} + \cancel{C_{Di}}\right]}{\ln \dfrac{C_{Bi} - C_{Di} - \frac{W}{Q_D}}{C_{Bi} - \frac{W}{Q_B} - C_{Di}}}$$

$$\cancel{W_{coc}} = \frac{KA\,\cancel{W}\left[\frac{1}{Q_B} - \frac{1}{Q_D}\right]}{\ln \dfrac{C_{Bi} - C_{Di} - \frac{W}{Q_D}}{C_{Bi} - C_{Di} - \frac{W}{Q_B}}}$$

$$\ln \frac{\cancel{(C_{Bi} - C_{Di})} \cdot \left[1 - \frac{W}{(C_{Bi} - C_{Di})} \cdot \frac{1}{Q_D}\right]}{\cancel{(C_{Bi} - C_{Di})} \left[1 - \frac{W}{(C_{Bi} - C_{Di})} \cdot \frac{1}{Q_B}\right]} = \frac{KA}{Q_B}\left[1 - \frac{Q_B}{Q_D}\right]$$

$$Z = \frac{Q_B}{Q_D} \qquad N_T = \frac{KA}{Q_B}$$

$$\ln \frac{\left[1 - \frac{D}{Q_B} \cdot \frac{Q_B}{Q_D}\right]}{\left[1 - E\right]} = N_T\left[1 - Z\right]$$

$$\ln \frac{[1-Ez]}{[1-E]} = N_T(1-z)$$

$$\frac{[1-Ez]}{[1-E]} = e^{N_T(1-z)}$$

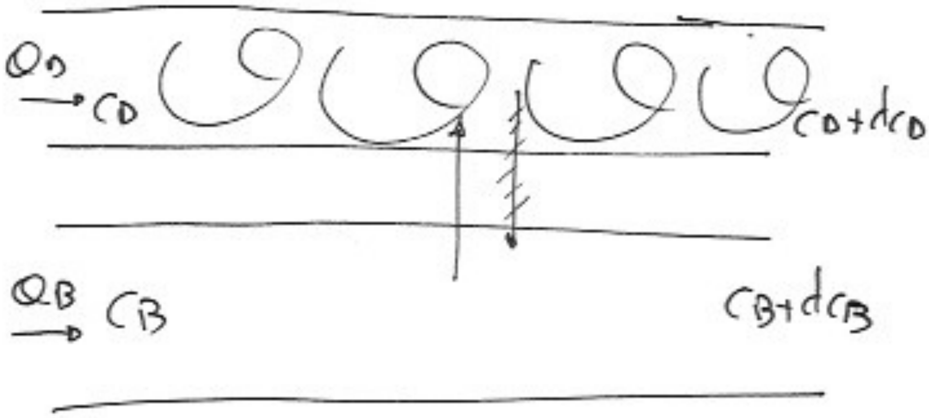
$$1-Ez = e^{N_T(1-z)} - E\,e^{N_T(1-z)}$$

$$E\left[e^{N_T(1-z)} - z\right] = e^{N_T(1-z)} - 1$$

$$E_{coc} = \frac{e^{N_T(1-z)} - 1}{e^{N_T(1-z)} - z} \qquad z = \frac{Q_B}{Q_D}$$

$$E_{coc} = \frac{e^{N_T} - 1}{e^{N_T}} = 1 - e^{-N_T}$$

# Flusso misto




$C_{Di} = C_{Do}$

$$W_{C.C.} = KA \frac{(C_{Bi} - C_{Di}) - (C_{Bo} - C_{Do})}{\ln \frac{(C_{Bi} - C_{Di})}{(C_{Bo} - C_{Do})}}$$

$$W_{f.A} = KA \frac{(C_{Bi} - C_{Do}) - (C_{Bo} - C_{Do})}{\ln \frac{(C_{Bi} - C_{Do})}{(C_{Bo} - C_{Do})}}$$

$$W = -Q_B (C_{Bo} - C_{Bi}) \Rightarrow -\frac{W}{Q_B} = C_{Bo} - C_{Bi}$$

$$\| \; C_{Bo} = C_{Bi} - \frac{W}{Q_B}$$

$$W = Q_D (C_{Do} - C_{Di}) \Rightarrow \frac{W}{Q_D} = C_{Do} - C_{Di}$$

$$\| \; C_{Do} = C_{Di} + \frac{W}{Q_D}$$

nella condizione di innesto — flusso

$$W_{F.n} = \frac{KA\,\left(C_{Bi} - \cancel{C_{Do}} - C_{Bo} + \cancel{C_{Do}}\right)}{\ln \dfrac{C_{Bi} - C_{Do}}{C_{Bi} - \dfrac{W}{Q_B} - C_{Do}}}$$

$$W = \frac{KA\left[\cancel{C_{Bi}} - \cancel{C_{Bi}} + \dfrac{W}{Q_B}\right]}{\ln \dfrac{(C_{Bi} - C_{Do})}{\left[C_{Bi} - C_{Do} - \dfrac{W}{Q_B}\right]}}$$

$$\ln \frac{\cancel{(C_{Bi} - C_{Do})}}{\cancel{(C_{Bi} - C_{Do})}\left[1 - \dfrac{W}{\underset{\overset{\shortparallel}{C_{Di}}}{(C_{Bi} - C_{Do})}} \cdot \dfrac{1}{Q_B}\right]} = \frac{KA}{Q_B}$$

$$\ln \frac{1}{1 - E} = N_T$$

$$1 - E = e^{-N_T}$$

$$E = 1 - e^{-N_T}$$

$$Z \simeq \phi = \frac{Q_B}{Q_D} \qquad Q_D \gg Q_B$$

$Q_D >> Q_B$

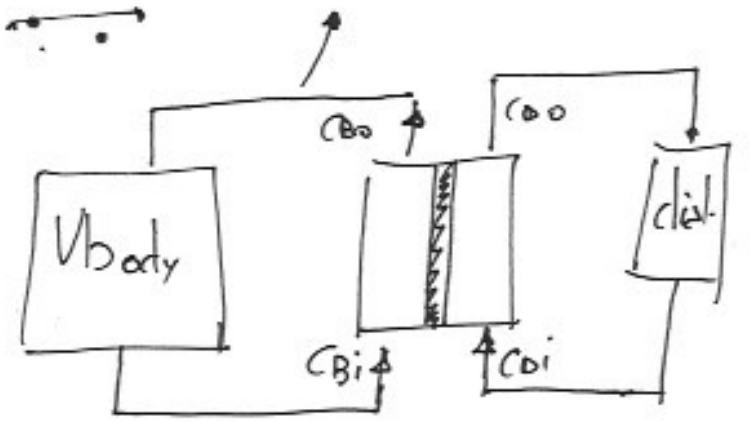
① $E = 1 - e^{-N_T} = 1 - e^{-\frac{KA}{Q_B}}$

$$E = \frac{D}{Q_B}\Big|_{C_{Di}=\phi} = \frac{\cancel{Q_B}(C_{Bi}-C_{Bo})}{(C_{Bi}-C_{Di})}\cdot\frac{1}{\cancel{Q_B}}\Big|_{C_{Di}=0}$$

② $E = \frac{C_{Bi}-C_{Bo}}{C_{Bi}} = 1 - \frac{C_{Bo}}{C_{Bi}}$

$$\cancel{1} - \frac{C_{Bo}}{C_{Bi}} = \cancel{1} - e^{-\frac{KA}{Q_B}}$$

$$C_{Bo} = C_{Bi}\, e^{-\frac{KA}{Q_B}}$$




• $\beta = e^{-\frac{KA}{Q_B}}$

$$V_b \frac{dC_{Bi}}{dt} = Q_B(C_{Bo}-C_{Bi}) = Q_B\left[C_{Bi}e^{-\frac{KA}{Q_B}} - C_{Bi}\right]$$

$$V_b \frac{dC_{Bi}}{dt} = Q_B C_{Bi}\left[e^{-\frac{KA}{Q_B}} - 1\right] = Q_B C_{Bi}[\beta - 1]$$

$$V_b \frac{dC_{Bi}}{dt} = Q_B\, C_{Bi}\, (\beta - 1)$$

$$\frac{dC_{Bi}}{C_{Bi}} = \frac{Q_B}{V_b} (\beta - 1)\, dt$$

$$\int_{C_{Bi}^{ti}}^{C_{Bi}^{tf}} \frac{dC_{Bi}}{C_{Bi}} = \frac{Q_B}{V_b} (\beta - 1) \int_{ti}^{tf} dt$$

$$\ln \frac{C_{Bi}^{tf}}{C_{Bi}^{ti}} = \frac{Q_B}{V_b} (\beta - 1)(tf - ti)$$

$$tf - ti = \tau$$

$$C_{Bi}^{tF} = C_{Bi}^{\phi}\, e^{\frac{(\beta - 1)\,\tau \cdot Q_B}{V_b}}$$

equazione whole body

$$C_{Bi}^{tF} = \frac{C_{Bi}^{\phi}}{2}$$

$$\frac{\cancel{C_{Bi}^{\phi}}}{2} = \cancel{C_{Bi}^{\phi}}\, e^{\frac{(\beta - 1) Q_B}{V_b} \cdot \tau}$$

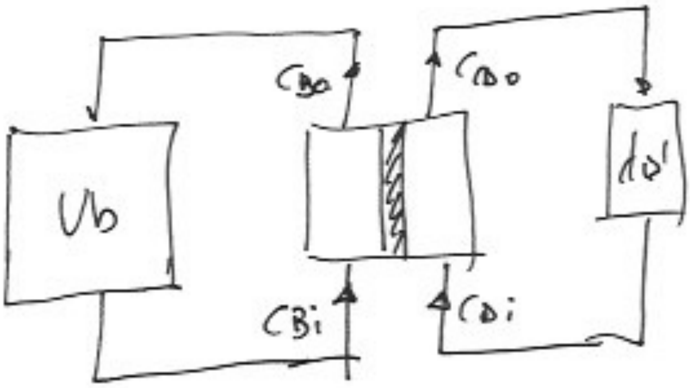
$$0.5 = e^{\frac{(\beta - 1) Q_B}{V_b} \cdot \tau} \qquad \frac{(\beta - 1) Q_B}{V_b} \cdot \tau = \ln 0.5$$

$$\tau_{1/2} = \frac{(\ln 0.5) \cdot V_b}{Q_B(\beta - 1)}$$

$\Rightarrow V_b = 5\text{-}6\ \ell$

$Q_B = 200 \frac{m\ell}{min}$

$\beta = e^{\frac{KA}{Q_B}}$




$$C_{Bi}^{tf} = C_{Bi}^{\phi}\, e^{\frac{(\beta - 1)\tau Q_B}{V_b}}$$

$$C_{Di}^{tf} = C_{Di}^{\phi}\, e^{\frac{Q_D \tau_D (1 - \beta)}{V_D}}$$

$$\beta = e^{\frac{KA}{Q_B}}$$

$C_{Di}^{\phi} = \phi$

Urea, acido: No — $Na^+$, glucosio

$$\tau_{dialisi} = \min(\tau_i)$$

Urea
AcidoUrico
$Na^+$
Glucosio

$$\left.\begin{matrix} C_{urea} \\ C_{acido\,urico} \end{matrix}\right\} = 0.1\, C_i$$