

# Allometry

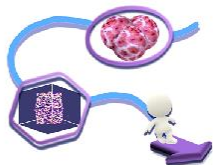
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# Allometric scaling $Y = aM^b$

- Known as the science of scaling in biology
- Has been studied since 1930
- Body parameters are related to mass through power laws



Parameter Y	b
Metabolism	3/4
Cell number	1
Flow rate	1/4



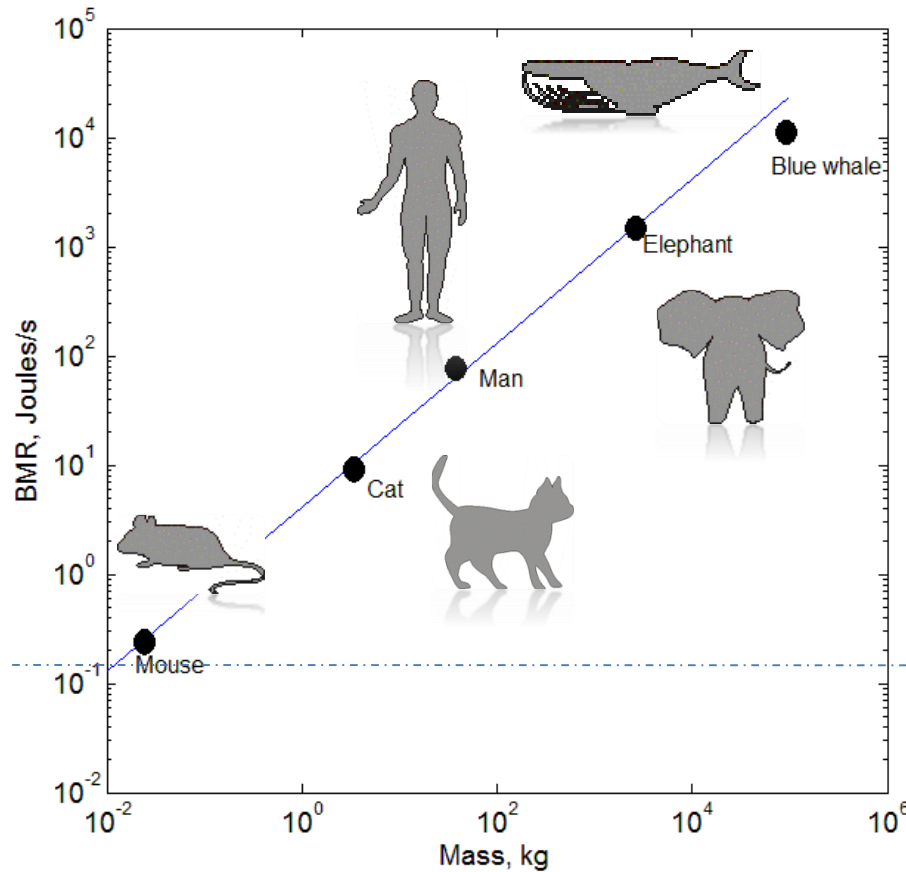
# Isometric Scaling

- Proportional scaling, i.e if mass doubles, mass of head, and other organs also doubles. If overall length increases, then all limbs increase proportionately..



## BMR / mass relationship, $b=3/4$

Y



M

$$Y = aM^b$$

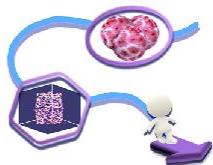
$$\log Y = b \log M + \log a$$

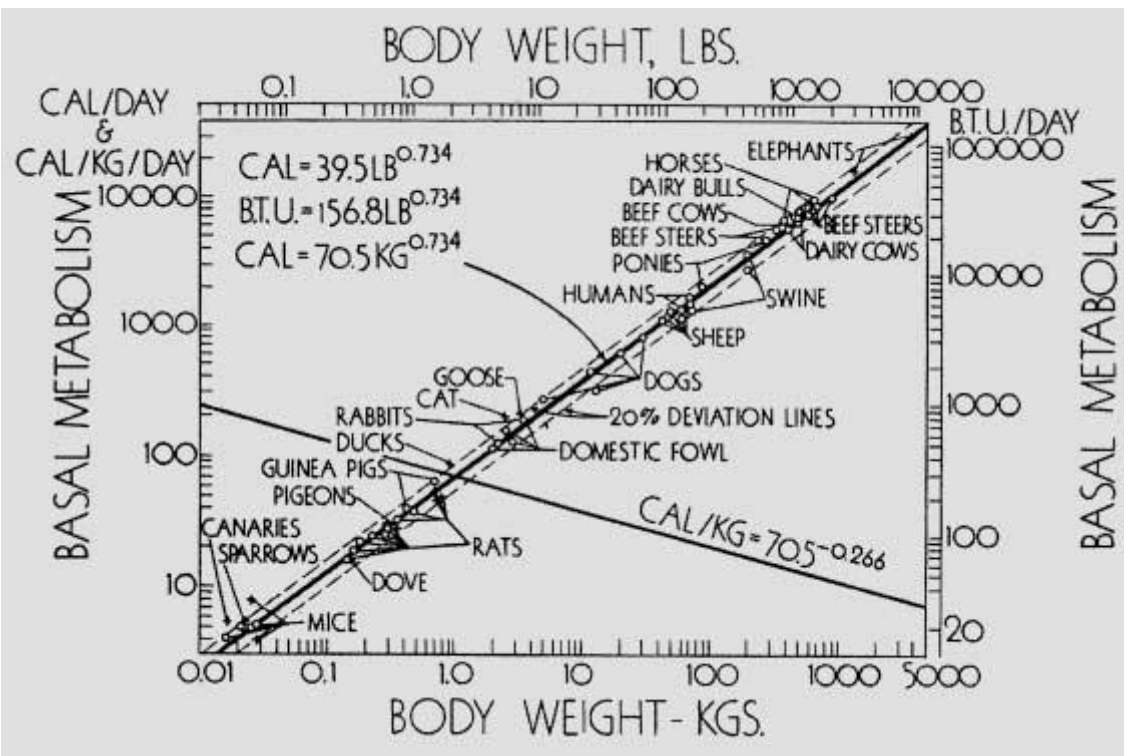
$b$  is the slope  
Log  $a$  the intercept



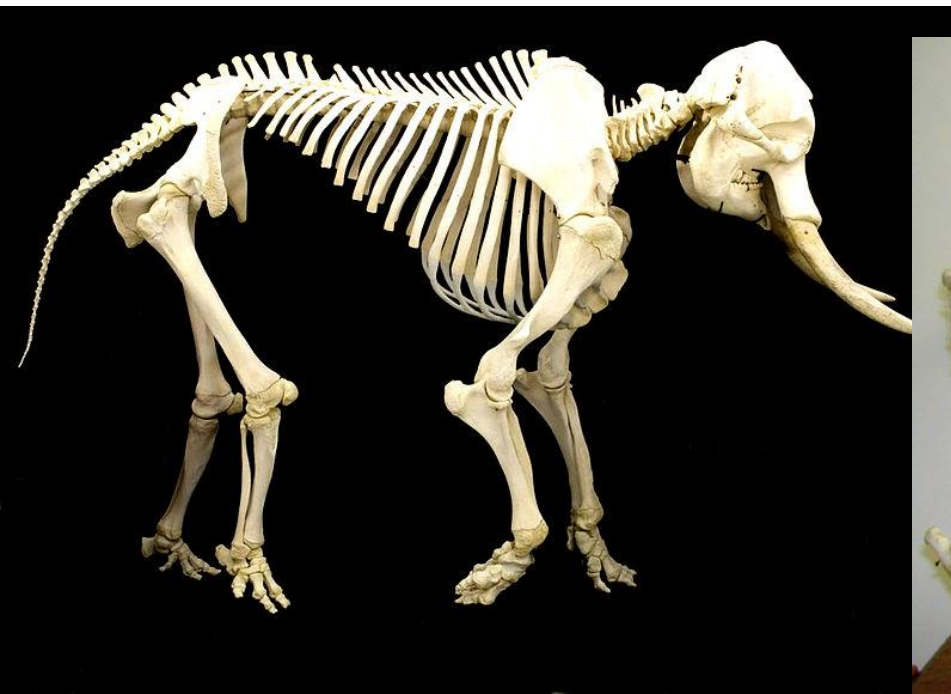
$$Y = aM^b$$

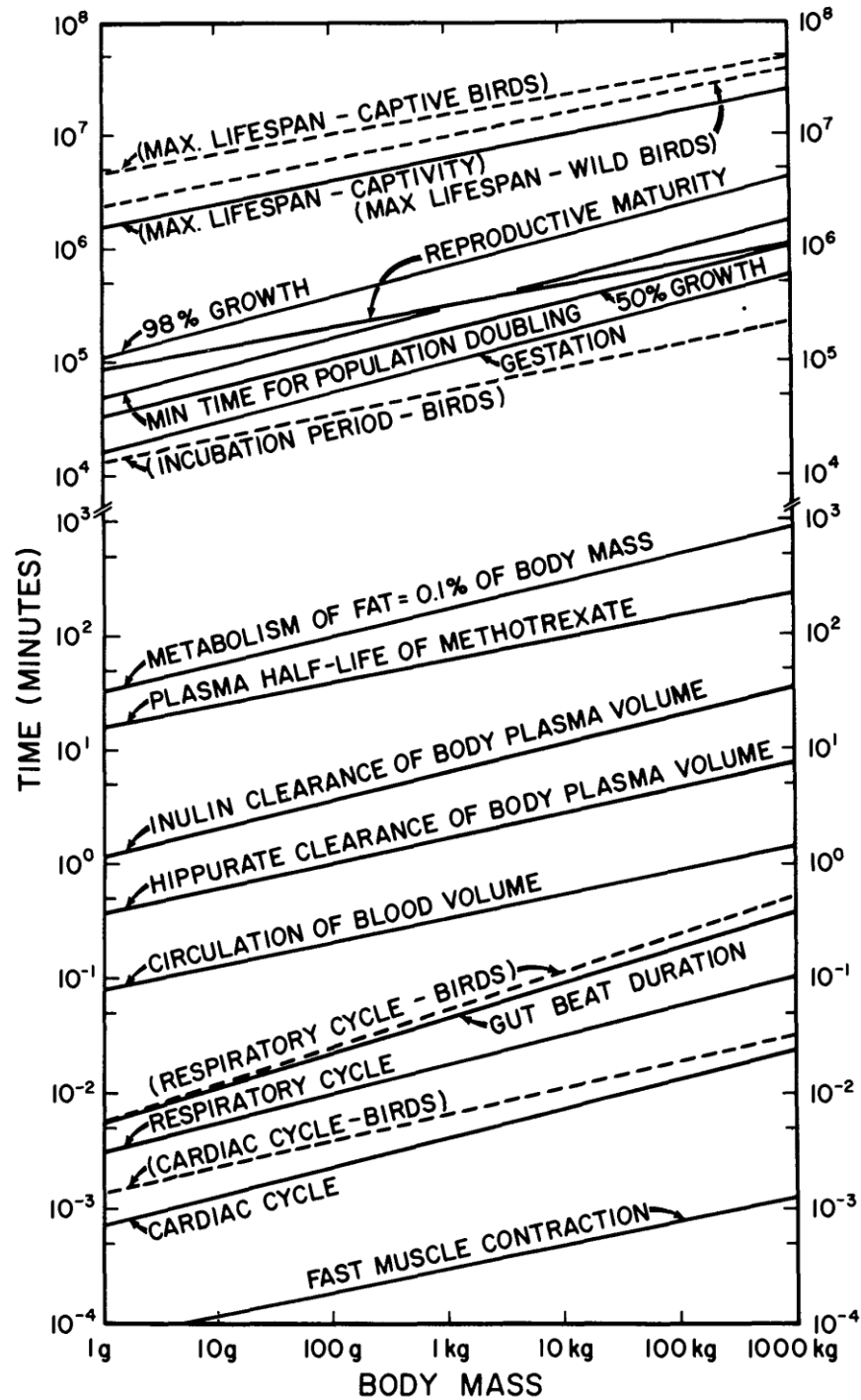
$b$	Significance	Example ( $b$ value)
0	Parameter does not change with body mass	Bone density in mammals, cell radius
1	Parameter changes in direct proportion with body mass	Body volume, cell number
$0 < b < 1$	Parameter increases at a slower rate than body mass	Metabolic rate (3/4), blood flow rate (3/4), external surface area (2/3), life span (1/4)
$> 1$	Parameter increases at a faster rate than body mass	Bone mass (4/3)
$< 0$	Parameter increases with body mass	Almost all frequencies or rates (cardiac frequency, respiratory frequency, -1/4)





# Bone mass





**Table 1.** Values of allometric exponents for variables of the mammalian cardiovascular and respiratory systems predicted by the model compared

with empirical observations. Observed values of exponents are taken from (2, 3); ND denotes that no data are available.

Cardiovascular			Respiratory		
Variable	Exponent		Variable	Exponent	
	Predicted	Observed		Predicted	Observed
Aorta radius $r_0$	$3/8 = 0.375$	0.36	Tracheal radius	$3/8 = 0.375$	0.39
Aorta pressure $\Delta p_0$	$0 = 0.00$	0.032	Interpleural pressure	$0 = 0.00$	0.004
Aorta blood velocity $u_0$	$0 = 0.00$	0.07	Air velocity in trachea	$0 = 0.00$	0.02
Blood volume $V_b$	$1 = 1.00$	1.00	Lung volume	$1 = 1.00$	1.05
Circulation time	$1/4 = 0.25$	0.25	Volume flow to lung	$3/4 = 0.75$	0.80
Circulation distance $l$	$1/4 = 0.25$	ND	Volume of alveolus $V_A$	$1/4 = 0.25$	ND
Cardiac stroke volume	$1 = 1.00$	1.03	Tidal volume	$1 = 1.00$	1.041
Cardiac frequency $\omega$	$-1/4 = -0.25$	-0.25	Respiratory frequency	$-1/4 = -0.25$	-0.26
Cardiac output $\dot{E}$	$3/4 = 0.75$	0.74	Power dissipated	$3/4 = 0.75$	0.78
Number of capillaries $N_c$	$3/4 = 0.75$	ND	Number of alveoli $N_A$	$3/4 = 0.75$	ND
Service volume radius	$1/12 = 0.083$	ND	Radius of alveolus $r_A$	$1/12 = 0.083$	0.13
Womersley number $\alpha$	$1/4 = 0.25$	0.25	Area of alveolus $A_A$	$1/6 = 0.083$	ND
Density of capillaries	$-1/12 = -0.083$	-0.095	Area of lung $A_L$	$11/12 = 0.92$	0.95
O <sub>2</sub> affinity of blood $P_{50}$	$-1/12 = -0.083$	-0.089	O <sub>2</sub> diffusing capacity	$1 = 1.00$	0.99
Total resistance $Z$	$-3/4 = -0.75$	-0.76	Total resistance	$-3/4 = -0.75$	-0.70
Metabolic rate $B$	$3/4 = 0.75$	0.75	O <sub>2</sub> consumption rate	$3/4 = 0.75$	0.76